Place Value & Radices

Base x

The base x number of 162.25:

Digit	1	6	2 •	2	5
Position	2	1	0	-1	-2
Value	$1x^2$	$6x^1$	$2x^0$	$2x^{-1}$	$5x^{-2}$

x is the "radix"
To find the value of a number, we do the sum of digit*radix^{position} for each digit.
We also call the radix a base

We typically use base 10, but we may use binary (base 2)
These work in the same way, 13 in this base is 1101.

So how would that work with base (–2)?
Simply put, we use the same digits as in base 2, but we use subtraction to form some numbers, 13 is shown below

Digit	1	1	1	0	1
Radix^{position}	16	-8	4	-2	1

And decimals?
o.5 is just 2^{-1}, and $(2^{-1})^{-1} = 2$, so we just flip it at the decimal point!

Surd bases

Take base $\sqrt{2}$, how would you expect that to work?
Well, given that $(\sqrt{x})^{2y} = x^y$, we can just use the regular base 2, but with a 0 between each digit, unless we want to display a multiple of $\sqrt{2}$ itself.

i?

i, or the imaginary unit is defined as $\sqrt{(-1)}$. This means that when we apply powers to it, we get alternations of (-𝕀),(-ℝ),(+𝕀), & (+ℝ). {where ℝ denotes a real number, and 𝕀 denotes an imaginary one.}
This results in the representation of13-13.5*i* in base 2*i* to be as below.

Digit	1	2	1	2	1 •	3
Radix^{position}	16	-8*i*	-4	2*i*	1	-0.5*i*

Base 1

Easily the simplest base, and one we've all probably used without thinking. Since 1^x is always just 1, for any value of x, all digits (represented by "|") represent +1. We often call this a "tally mark". 13 in this base is ||||||||||||.

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i, (https://www.oed.com/search/dictionary/?q=imaginary+number&tl=true)
Unary, (https://oeis.org/wiki/Natural_numbers,_in_bases_1_through_10#cite_ref-1 {Notes section})
Radices, (pp. 13–14. ISBN 978-1-292-02468-4)
Negative Integer Bases, (https://brilliant.org/wiki/negative-integer-number-base/)