

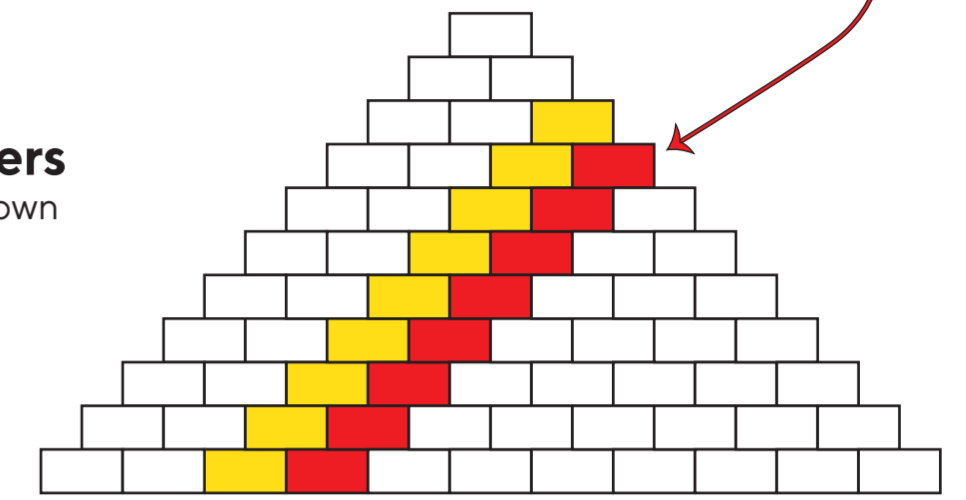
Al-Karaji's Triangle

Who knew!

Within this simple triangle lay many, many mathematical wonders. Here are just a few....

The Tetrahedral Numbers

In the red bricks live the tetrahedral numbers. Can you draw the pattern they make?

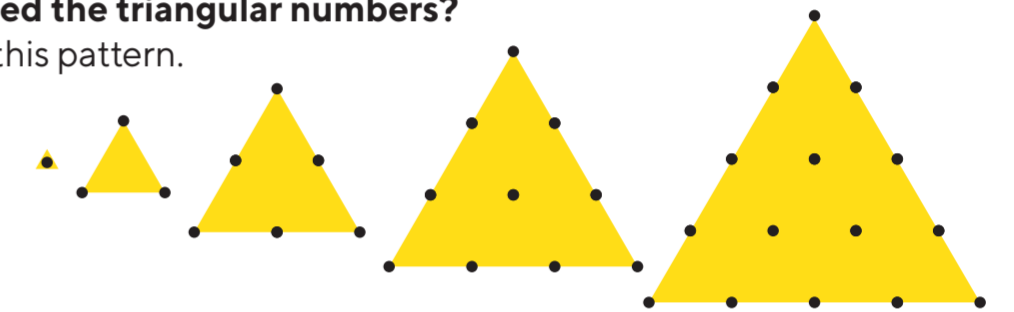


The Triangular Numbers

If you follow the yellow bricks down through each layer, it reveals the triangular numbers.

Do you know why they are called the triangular numbers?

Take a look at this pattern.



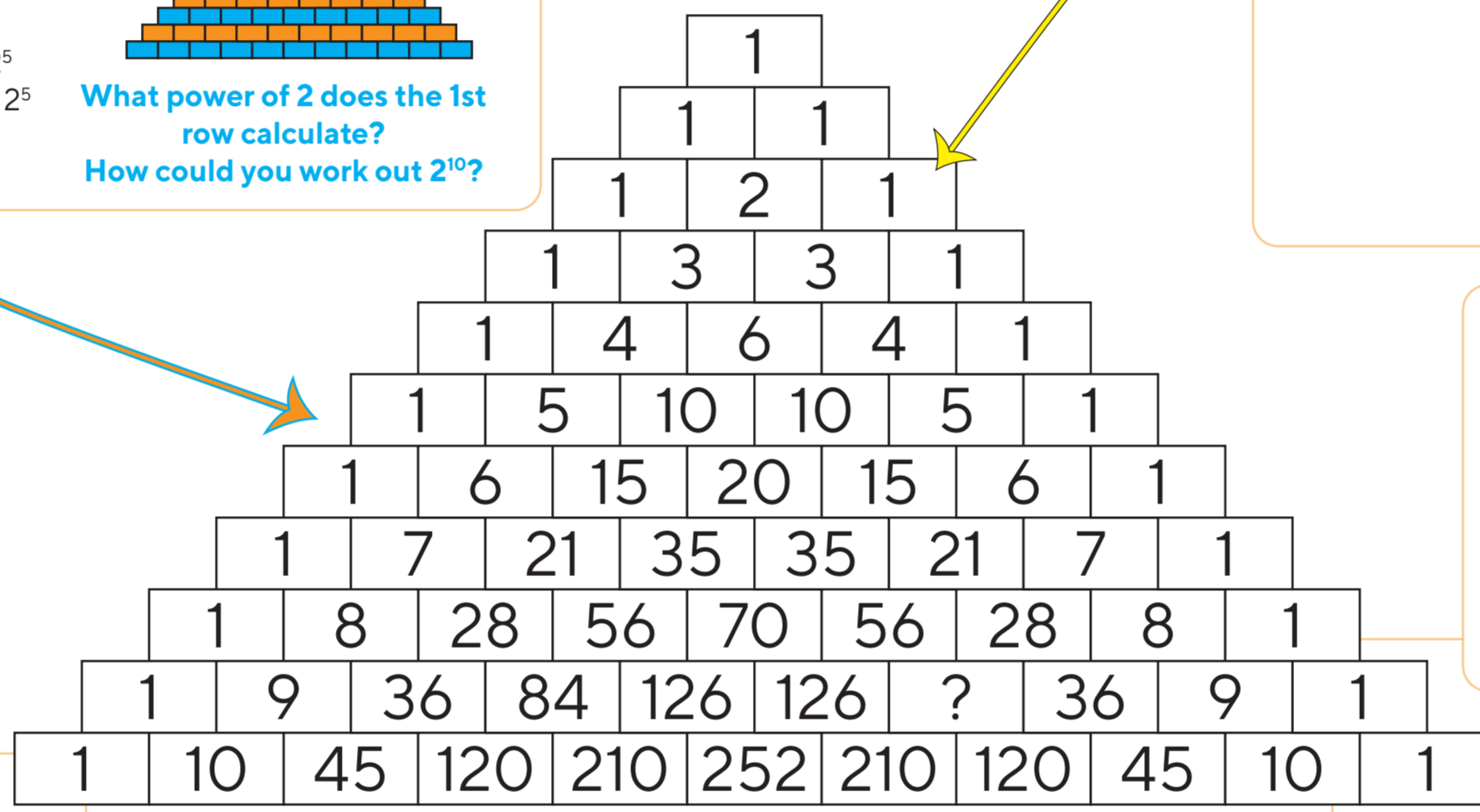
The Persian mathematician **Al-Karaji** wrote a book detailing the first formulation of this triangle in the 10th century. Other mathematicians from different cultures have also written about this triangle, including the Persian mathematician **Omar Khayyam** and Chinese mathematician **Jia Xian** in the 11th century. In Western culture, **Blaise Pascal** was the French Mathematician whom this triangle is often named after. With 1s in all the bricks on the sloping sides, all other bricks are the sum of the two bricks which touch its upper edge. **What value should be in the ? brick?**

The Powers of Two

If you calculate the total of each row a special set of numbers is revealed.

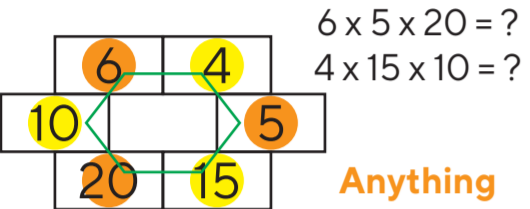
The 6th row calculates 2^5
 $1 + 5 + 10 + 10 + 5 + 1 = 32 = 2^5$

What power of 2 does the 1st row calculate?
 How could you work out 2^{10} ?



Hexagons

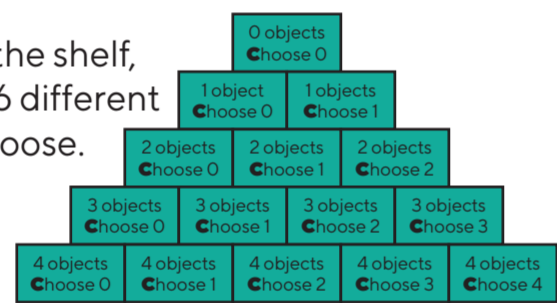
Calculate the product of the alternating vertices of any hexagon within Al-Karaji's Triangle.



Combinatorics

If we have a set of objects and you wish to choose a certain number of them, this triangle tells you how many possible ways there are to do this.

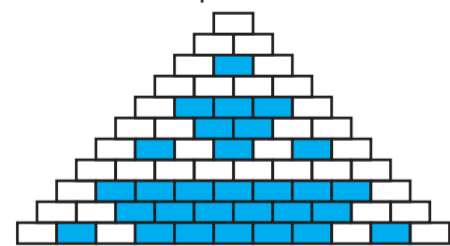
E.g. if you have 4 books on the shelf, I blindly pick any 2, there are 6 different pairs of books I could choose.



If you have 6 books and wish to choose 3 of them, how many different combinations are there?

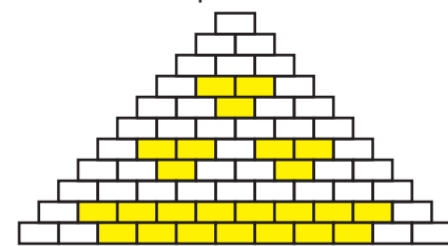
Multiples

If you shaded all the multiples of 2



What patterns do you create with other multiples? Try 4, 5, 6, 7,

If you shaded all the multiples of 3



Binomials

An algebraic expression with 2 parts is called a binomial. If raised to different integer (whole number) powers, we can use Al-Karaji's triangle to help us determine the coefficients.

$$(x + y)^0 = 1$$

$$(x + y)^1 = 1x + 1y$$

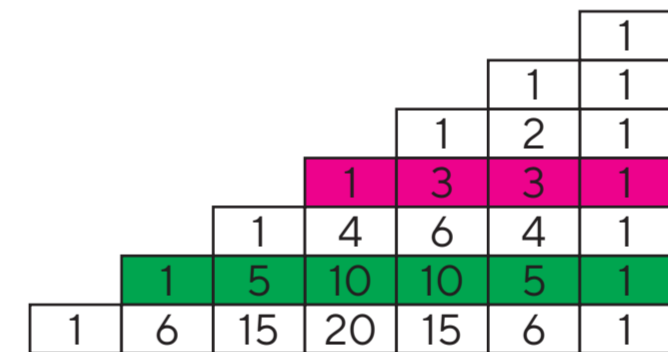
$$(x + y)^2 = 1x^2 + 2xy + 1y^2$$

$$(x + y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3$$

$$(x + y)^4 = 1x^4 + __ + __ + __ + 1y^4$$

Can you expand the binomial to the power of 4 using the triangle to help you?

10^6 10^5 10^4 10^3 10^2 10^1 10^0



The Powers of Eleven

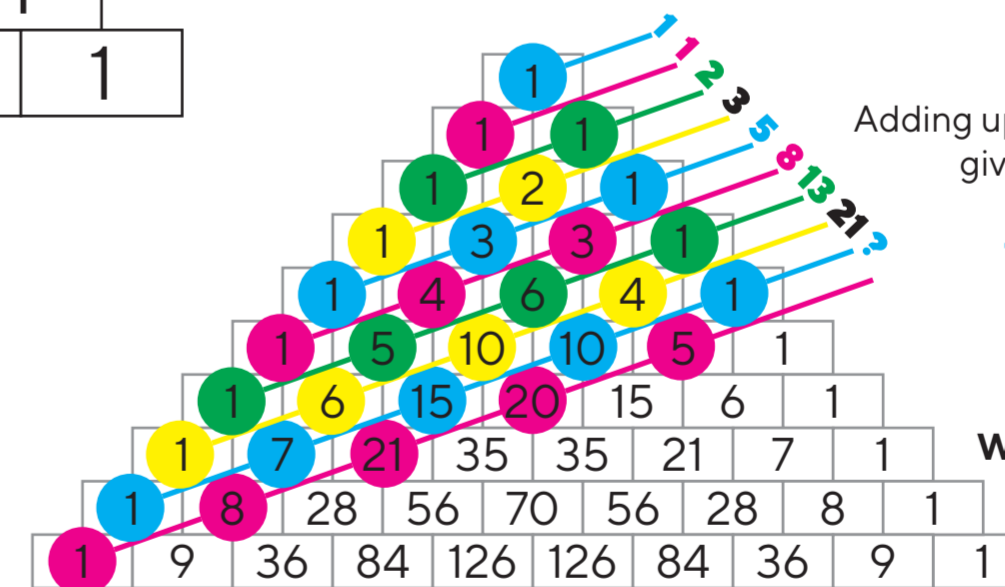
Right align the bricks in the triangle. Each row gives you a power of 11... $11^0 = 1, 11^1 = 11, 11^2 = 121$ etc..

What is 11^3 ?
 Can you figure out 11^5 ? Be careful!

Fibonacci

Adding up along these coloured diagonals gives you the numbers in the Fibonacci sequence

1, 1, 2, 3, 5, 8, 13, 21, ...



What's next? Is there any other way to determine the next number?

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Trigonometry

The word **trigonometry** has evolved from **trigonon** meaning three (tri) angles (gon) and **metria** meaning to measure.

Imagine
If we take this right angled triangle and enlarge it and shrink it, to lots of different sizes, keeping all the sides in proportion....

What do you notice?
Did you spot all the angles stayed the same.

A Different Angle...?
If we change the size of the original angle in the triangle, and do some more enlargements, again the angle stays the same in all of the new triangles we create.

Give them a name!
Naming the sides, depending on where they are in relation to the angle, helps us to calculate things.

In each of these three triangles, divide the Opposite by the Hypotenuse.

$\frac{8}{12} = \frac{2}{3}$ $\frac{5}{7.5} = \frac{2}{3}$ $\frac{10}{15} = \frac{2}{3}$

In each of these three triangles, divide the Adjacent by the Hypotenuse.

$\frac{12}{15} = \frac{4}{5}$ $\frac{4}{5}$ $\frac{6}{7.5} = \frac{4}{5}$

So
When the angle stays the same, all the sides stay in the same proportions.

To 360° and beyond
We can continue to increase the angle and move the triangle around the origin, if we continue to plot the height of the triangle the graph below is drawn.

What do you think happens as the angle exceeds 360°?

The Sine Wave **sin** **cos** **tan**

This function is called the Sine function (shortened to **sin**). It carries on forever, as you continue around and around the circle. Luckily we have a button on the calculator which stores this graph and determines its values when we need them.

If Sine, relates to the opposite and hypotenuse sides, how do you think Cos(ine) and Tan(gent) are related?

Bigger than 90°
If we draw our triangles in a unit circle, (often, in maths **unit** is a word used for 1, so a **unit circle** has a radius of 1). As the hypotenuse is 1, the height of the triangle represents the ratio of the opposite and hypotenuse. If we define the angle as always being measured from the positive horizontal axis we can move the triangle around the origin.

As the angle gets closer to 90°, the height of the triangle cannot exceed 1.

For any given angle
The ratio of any two sides is always the same. This is because when the angle is fixed we know all the triangles are similar, and therefore sides must be in proportion.
When we find the ratio of any two sides for a fixed angle, we will always get the same answer. If we change the angle the ratio will change.

Let's plot it
If we draw a graph of the different angles on the horizontal axis and the ratio of the opposite and hypotenuse on the vertical axis, we get:

As the angle gets closer to 90°, the ratio seems to get no higher than 1.

What do you think happens to the graph with angles over 90°? Would it still be a right angled triangle?

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¹<https://www.wjec.co.uk/media/invh1fni/gce-a-level-provisional-results-june-2022.pdf>

Α Β Γ Δ Ε Ζ Η Θ Ι Κ Λ Μ Ν Ξ Ο Π Ρ Σ Τ Υ Φ Χ Ψ Ω

Quadrare

Quadrare

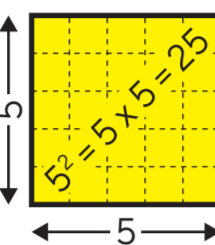
Is a Latin word meaning 'to make square', it's where the words **quadratics**, **quadrangle**, **quadrilateral**, **quadriceps**... come from.
Can you think of any more quad words?

Not everything is perfect

Just like in the real world, not everything in maths can be perfect - although much of maths is pretty good! There are lots of expressions that cannot be written as a perfect square

Make a square

When we square a number we can show this in a picture, it makes a square!



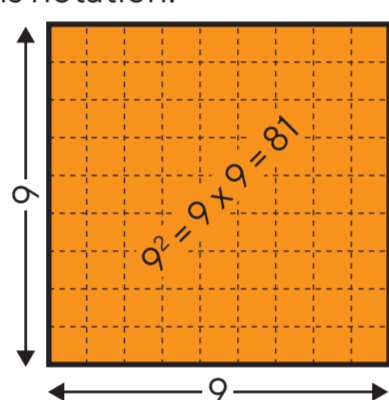
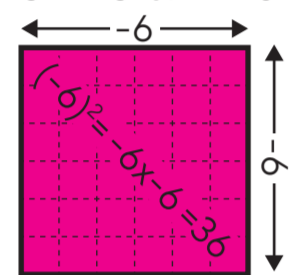
Squaring a number

When we square a number, we multiply by itself.

$$5 \times 5 = 25 \text{ or } 9 \times 9 = 81 \text{ or } -6 \times -6 = 36$$

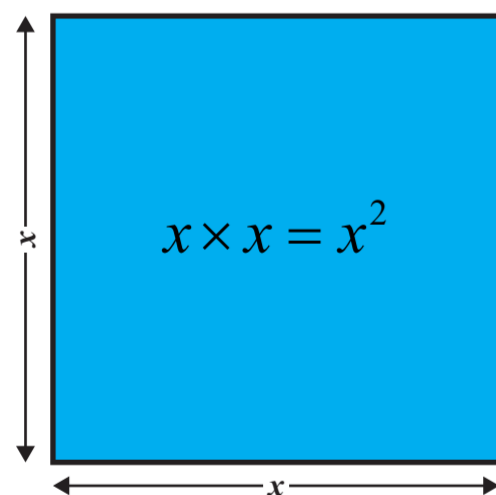
We use a superscript 2 to be the symbol for this operation. René Descartes developed this notation.

$$5^2 = 25 \text{ or } 9^2 = 81 \text{ or } (-6)^2 = 36$$



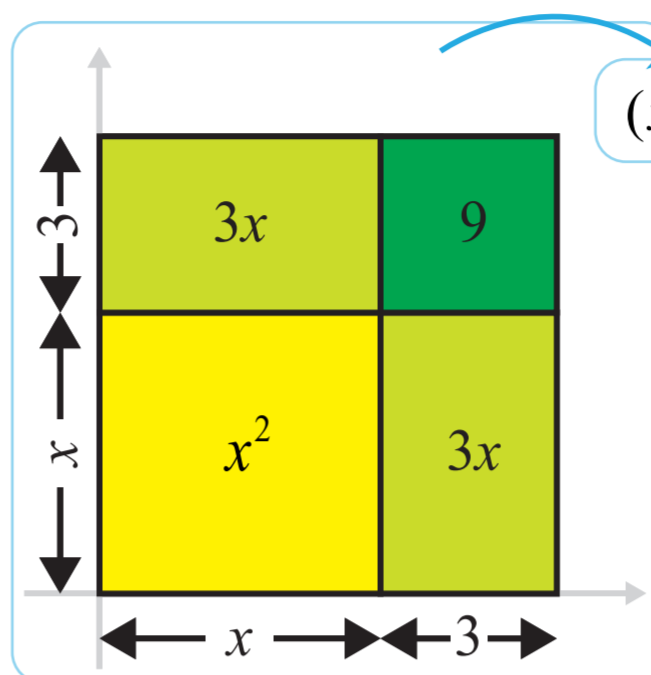
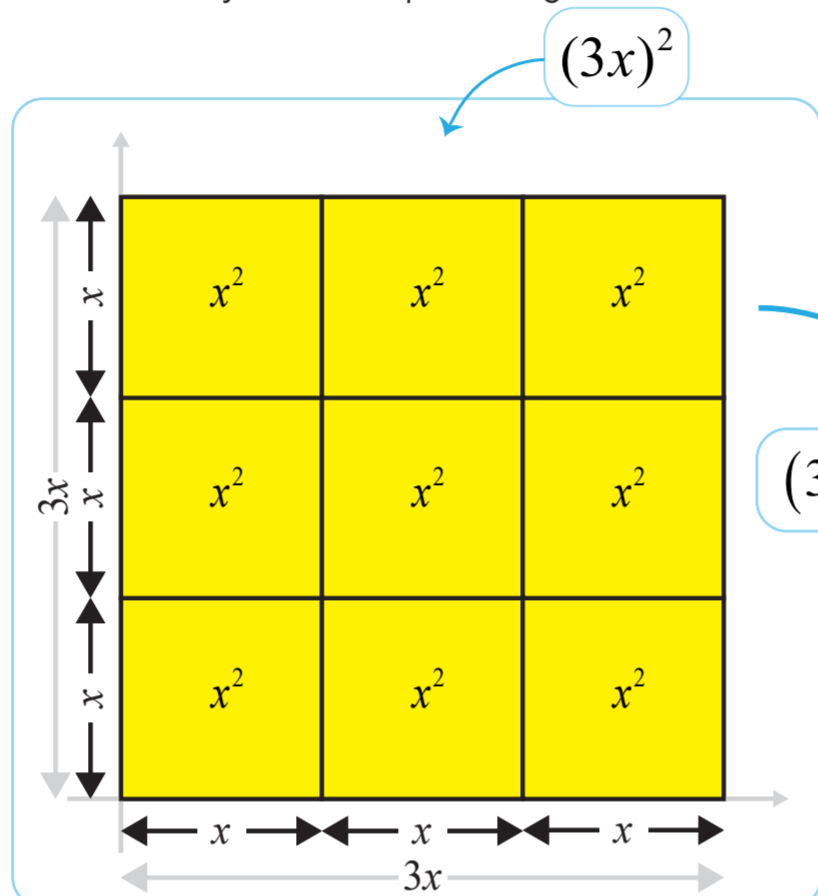
Squaring an unknown

When we square a number, we don't know yet we use a letter to represent this number. It could be any letter from any alphabet, quite often it's x , thanks to Descartes again for this!

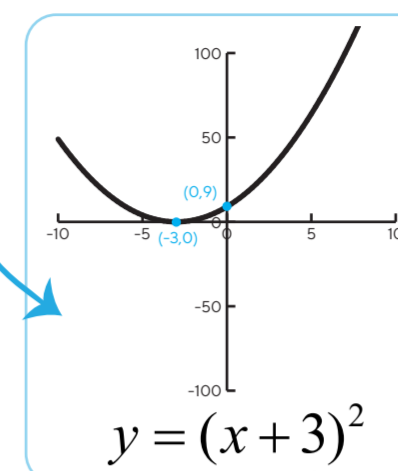


Squaring an expression

Pretty much anything can be squared. And we can always draw a square diagram to show it.



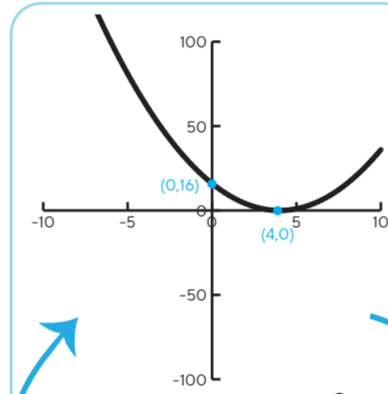
$$(x+3)^2 = x^2 + 6x + 9$$



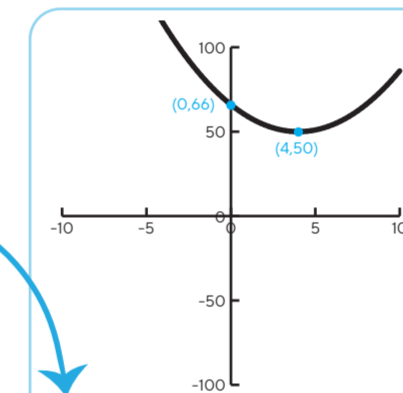
The curve touches the x axis at -3 . Can you think why?

Translations #1

When the graph of a perfect square quadratic function is drawn, it is a translation in the x direction.



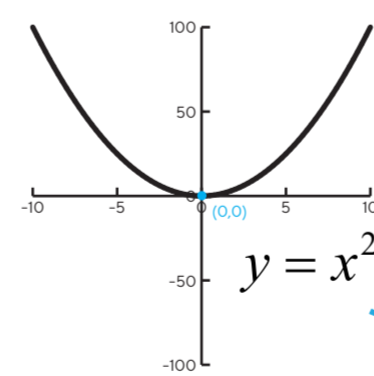
What's **different**? Why? What's the **same**? Why?



$$(x-4)^2 = x^2 - 8x + 16$$

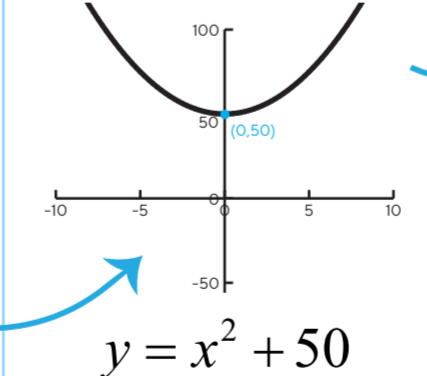
Plot it!

Plotting the simplest quadratic...



Huh?

What's **different**? Why? What's the **same**? Why?

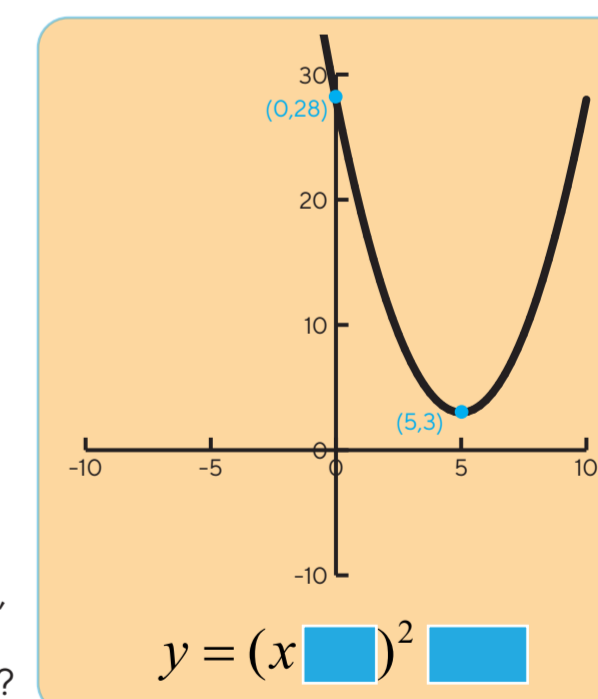


Translations #2

What has caused the vertical translation in the y direction? Can you see?

What is it?

By looking at the graph to the right, can you determine its equation. What goes in the blue boxes below?



Perfectly Square

Anything that can be written as something squared is called a **perfect square**. All of these are **perfect squares**:

$$(x+3)^2 \quad (3x)^2 \quad (x-4)^2$$

$$7^2 \quad (x+y+12-5z)^2$$

Quadratics!

An expression which has unknowns to a highest power of 2, and maybe some unknowns and possibly a constant, is called a Quadratic. No other powers allowed.

These are all quadratic expressions:

$$(x+3)^2 \quad 12 - r^2 + 3r \quad x^2 + 42$$

$$(t-10)^2 - 5 \quad x^2 \quad p^2 + p - 12$$

Answer: $y = (x-5)^2 + 3$

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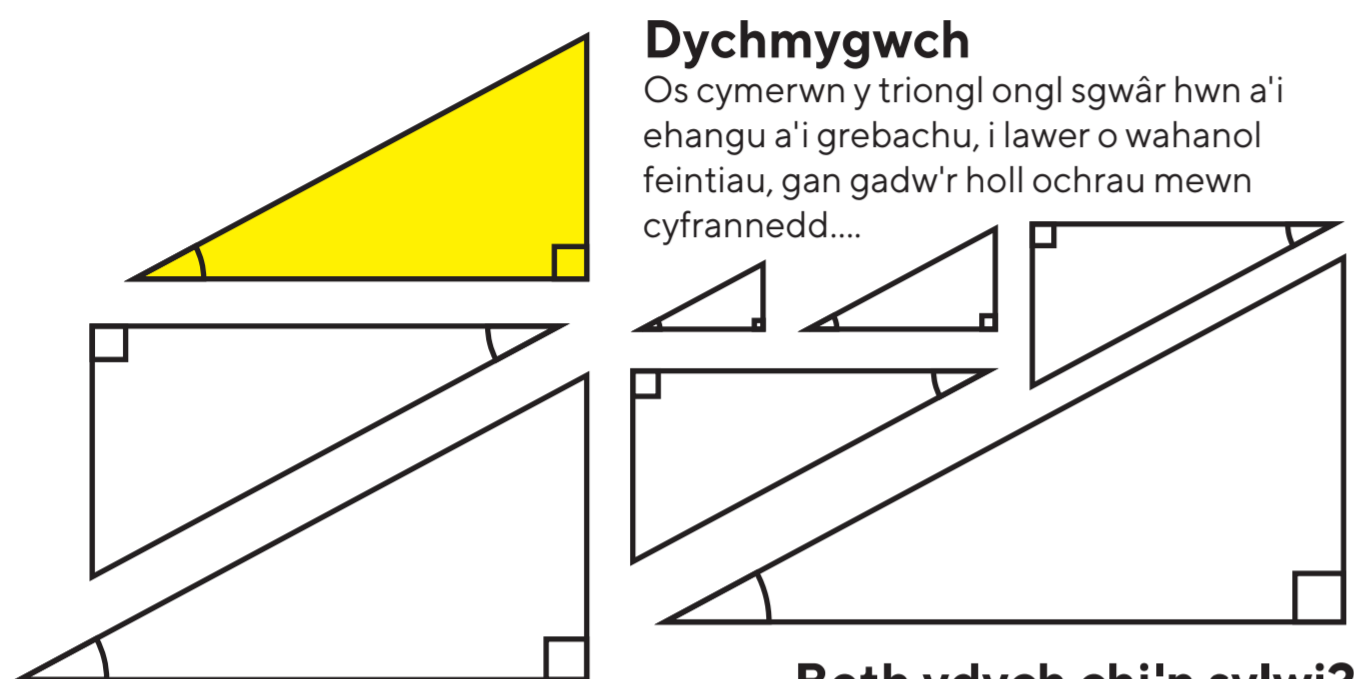
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Trigonometreg

Mae'r gair **trigonometreg** wedi esblygu o **trigonon** sy'n golygu tair (tri) ongl (gon) a **metria** sy'n golygu mesur.

Dychmygwch

Os cymerwn y tri ongl ongl sgwâr hwn a'i ehangu a'i grebachu, i lawer o wahanol feintiau, gan gadw'r holl ochrau mewn cyfrannedd....

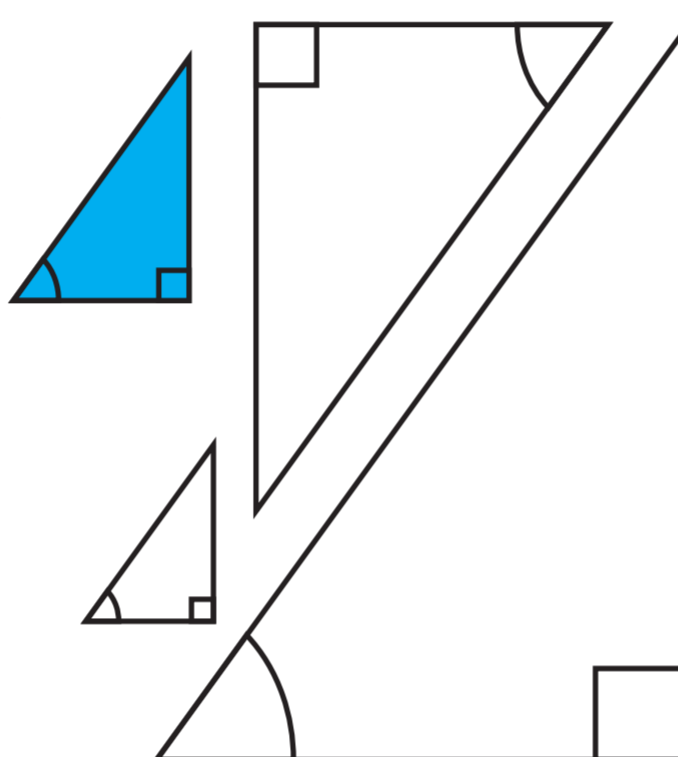


Beth ydych chi'n sylwi?

Oeddech chi'n sylwi bod yr holl onglau wedi aros yr un fath?

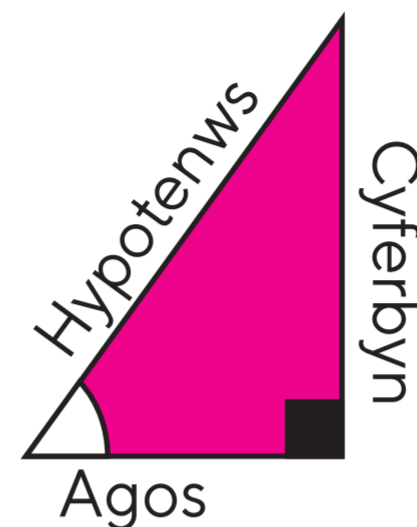
Ongl wahanol...?

Os ydym yn newid maint yr ongl wreiddiol yn y tri ongl, ac yn gwneud mwy o ehangiadau, eto mae'r ongl yn aros yr un fath ym mhob un o'r tri onglau newydd rydyn ni'n eu creu.



Rhowch enw iddyn nhw!

Mae enwi'r ochrau, yn ddiabynnol ar ble maen nhw mewn perthynas â'r ongl, yn ein helpu ni i gyfrifo pethau.

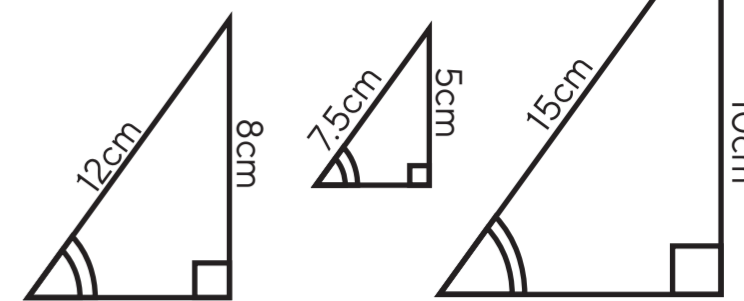


Mae'r ochr agos nesaf i'r ongl dan sylw.

Felly

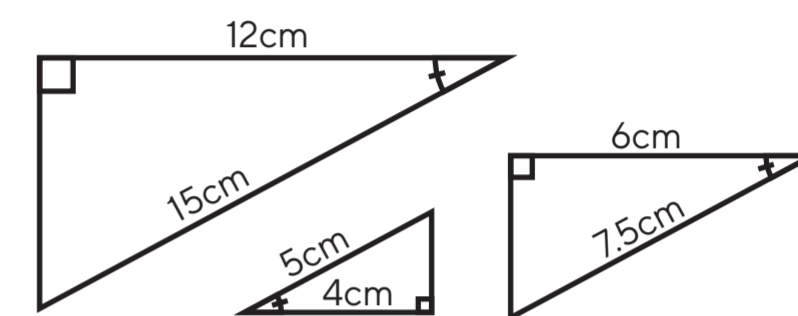
Pan fydd yr ongl yn aros yr un fath, mae'r holl ochrau'n aros yn yr un cyfrannedd.

Ym mhob un o'r tri thri ongl hyn, rhannwch y Cyferbyn â'r Hypotenws.



$$\frac{\text{Cyferbyn}}{\text{Hypotenws}} = \frac{8}{12} = \frac{2}{3} \quad = \frac{5}{7.5} = \frac{2}{3} \quad = \frac{10}{15} = \frac{2}{3}$$

Ym mhob un o'r tri thri ongl hyn, rhannwch yr Agos â'r Hypotenws.

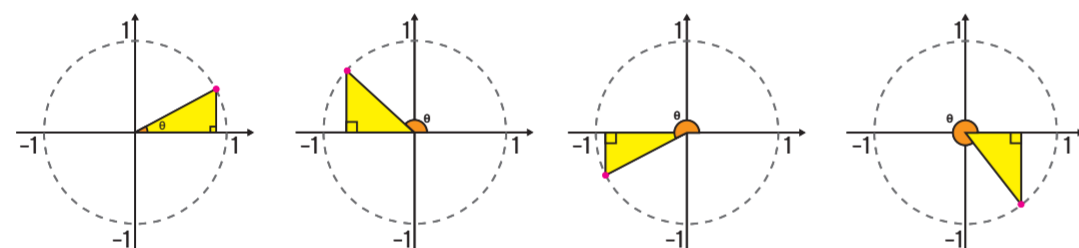


$$\frac{\text{Agos}}{\text{Hypotenws}} = \frac{12}{15} = \frac{4}{5} \quad = \frac{4}{5} \quad = \frac{6}{7.5} = \frac{4}{5}$$

Mae'r ochr gyferbyn yn ddirgroes i'r ongl dan sylw.

360° a thu hwnt

Gallwn barhau i gynyddu'r ongl a symud y tri ongl o amgylch y tardd, os ydym yn parhau i blotio uchder y tri ongl y mae'r graff isod yn cael ei lunio.

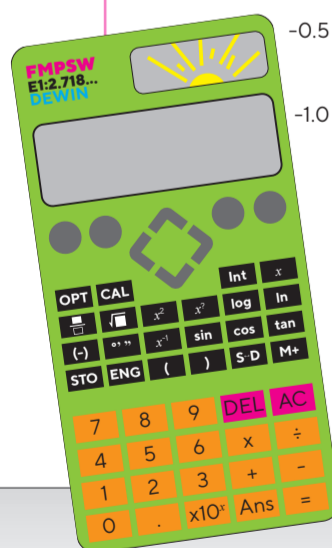


Beth ydych chi'n meddwl sy'n digwydd wrth i'r ongl fod yn fwy na 360°?

sin cos tan

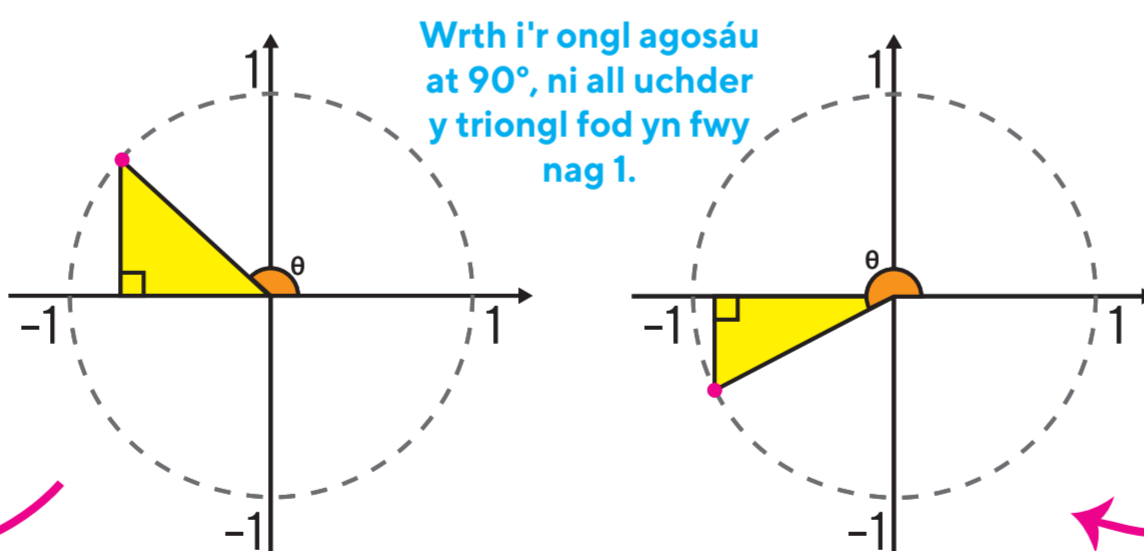
Y Don Sin

Gelwir y ffwythiant hwn yn ffwythiant Sin. Mae'n mynd ymlaen am byth, wrth i chi barhau o gwmpas ac o amgylch y cylch. Yn ffodus, mae gennym fotwm ar y cyfrifiannell sy'n storio'r graff hwn ac yn penderfynu ei werthoedd pan fydd eu hangen arnom.



Yn fwy na 90°

Os ydym yn tynnu ein tri onglau mewn cylch uned, (yn aml, ym mathemateg mae uned yn air a ddefnyddir ar gyfer 1, felly mae gan gylch uned radiws o 1). Gan fod yr hypotenws yn 1, mae uchder y tri ongl yn cynrychioli cymhareb y cyferbyn a'r hypotenws. Os ydym yn diffinio'r ongl fel bob amser yn cael ei mesur o'r echelin llorweddol positif, gallwn symud y tri ongl o amgylch y tardd.



Wrth i'r ongl agosáu at 90°, ni all uchder y tri ongl fod yn fwy nag 1.

Ar gyfer unrhyw ongl benodol

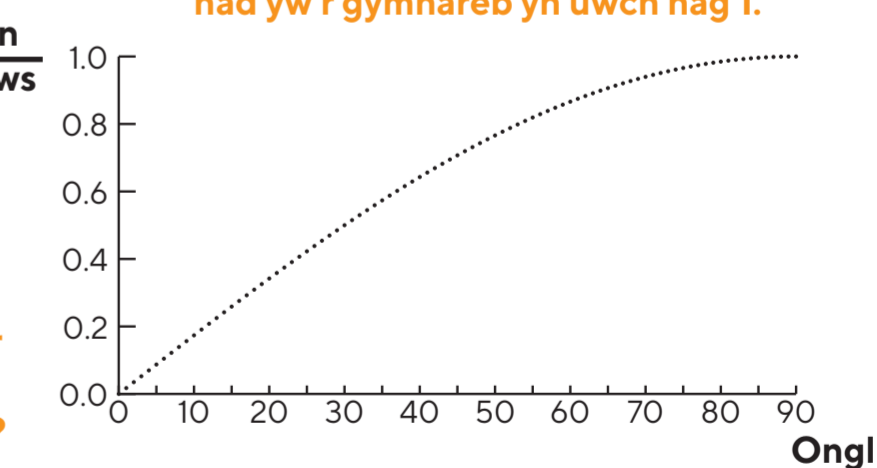
Mae'r gymhareb o unrhyw ddwy ochr bob amser yr un fath. Mae hyn oherwydd pan fydd yr ongl yn sefydlog rydym yn gwybod bod yr holl dri onglau yn gyflun, ac felly mae'n rhaid i'r ochrau fod mewn cyfrannedd.

Pan fyddwn yn darganfod cymhareb unrhyw ddwy ochr ar gyfer ongl sefydlog, byddwn bob amser yn cael yr un ateb. Os byddwn yn newid yr ongl bydd y gymhareb yn newid.

Gadewch i ni ei blotio

Os tynnwn graff o'r onglau gwahanol ar yr echelin llorweddol a chymhareb y cyferbyn a'r hypotenws ar yr echelin fertigol, fe gawn:

Cyferbyn Hypotenws



Wrth i'r ongl agosáu at 90°, ymddengys nad yw'r gymhareb yn uwch nag 1.

Beth ydych chi'n meddwl sy'n digwydd i'r graff gydag onglau dros 90°? A fyddai'n dal i fod yn dri ongl ongl sgwâr?

Mae mwy i Fathemateg nag ydych chi'n feddwl.... ymwelwch â rhgmc-mspw.cymru i ddarganfod mwy.

Gellir astudio **Lefel 2 Mathemateg Ychwanegol** yn ystod cyfnod allweddol pedwar.

Yng nghyfnod allweddol pump **Safon Uwch Mathemateg** yw'r **Safon Uwch** mwyaf poblogaidd¹ a **Mathemateg Bellach Safon Uwch** yw'r cydymaith perffaith.

Mae Rhaglen Gymorth Mathemateg Cymru (RhGMC) yma i gefnogi myfyrwyr, athrawon ac adrannau ledled Cymru wrth gyfoethogi a datblygu eu parth Mathemategol ar draws pob cyfnod allweddol.



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Quadrare

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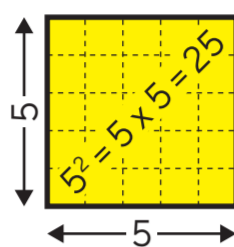
Mae'n air Lladin sy'n golygu 'i wneud yn sgwâr', dyma ble mae'r geiriau **cwadratig**, **cwadriceps**... yn dod o.
Allwch chi feddwl am fwy o eiriau cwad?

Nid yw popeth yn berffaith.

Yn union fel yn y byd go iawn, ni all popeth ym mathemateg fod yn berffaith - er bod llawer o fathemateg yn eithaf da! Mae yna lawer o fynegiadau na ellir eu hysgrifennu fel sgwâr perffaith

Llunio sgwâr

Pan fyddwn yn sgwario rhif gallwn ddangos hyn mewn llun, mae'n gwneud sgwâr!



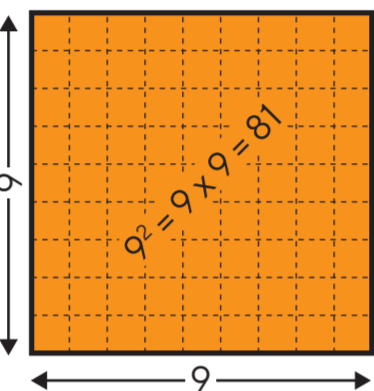
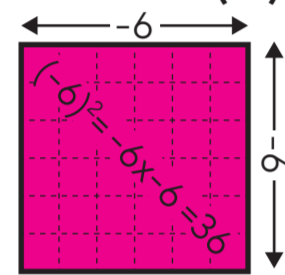
Sgwario rhif

Pan fyddwn yn sgwario rhif, rydym yn ei luosi ag ef ei hun.

$5 \times 5 = 25$ or $9 \times 9 = 81$ or $-6 \times -6 = 36$

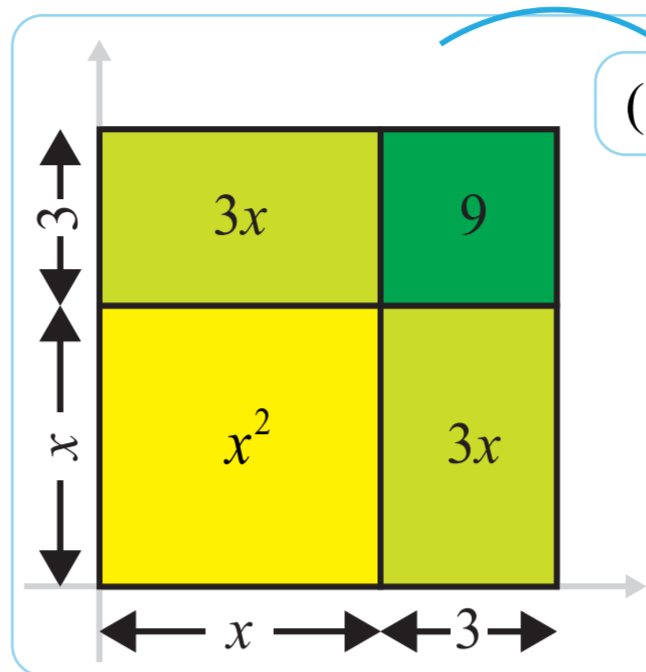
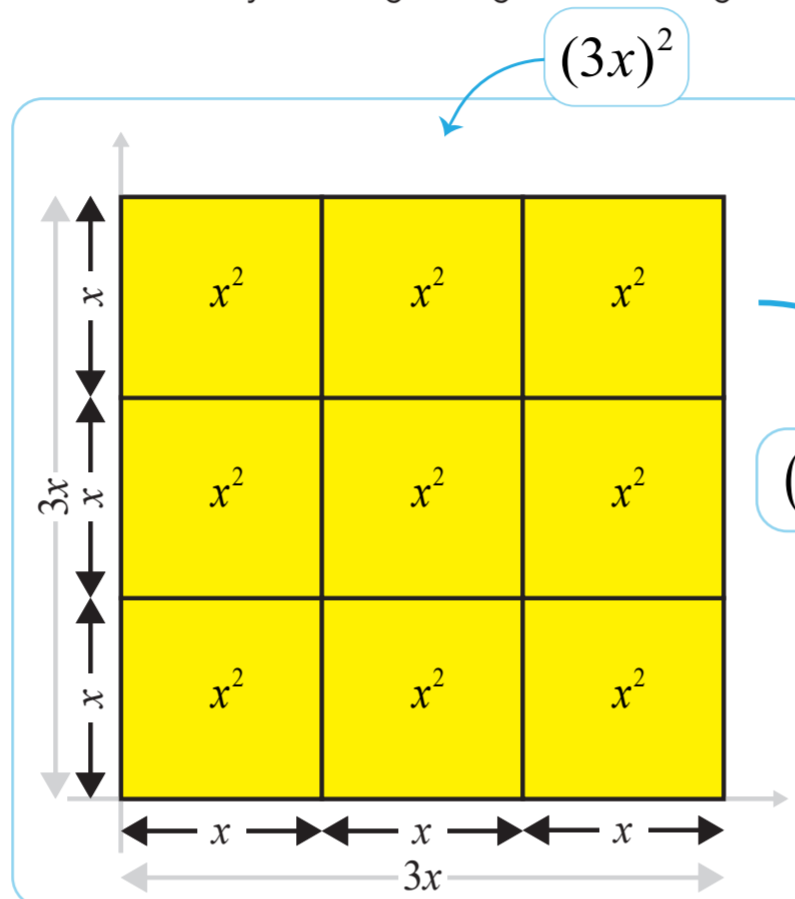
Rydym yn defnyddio uwchysgrif 2 i fod yn symbol ar gyfer y gweithrediad hwn. Datblygodd René Descartes y nodiant hwn.

$5^2 = 25$ neu $9^2 = 81$ neu $(-6)^2 = 36$

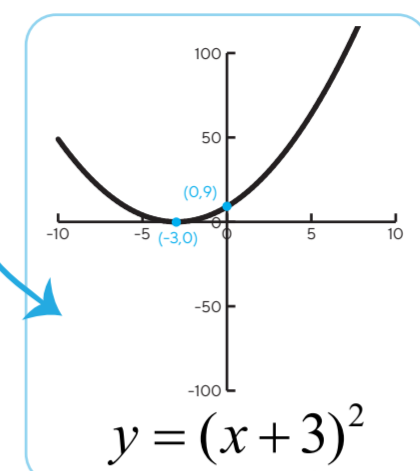


Sgwario mynegiad

Gall bron iawn unrhyw beth gael ei sgwario. A gallwn bob amser dynnu diagram sgwâr i'w ddangos.



$(x+3)^2 = x^2 + 6x + 9$

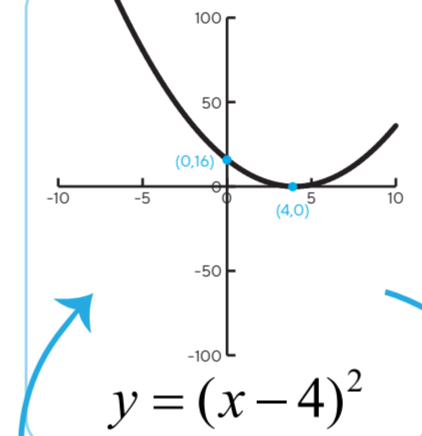
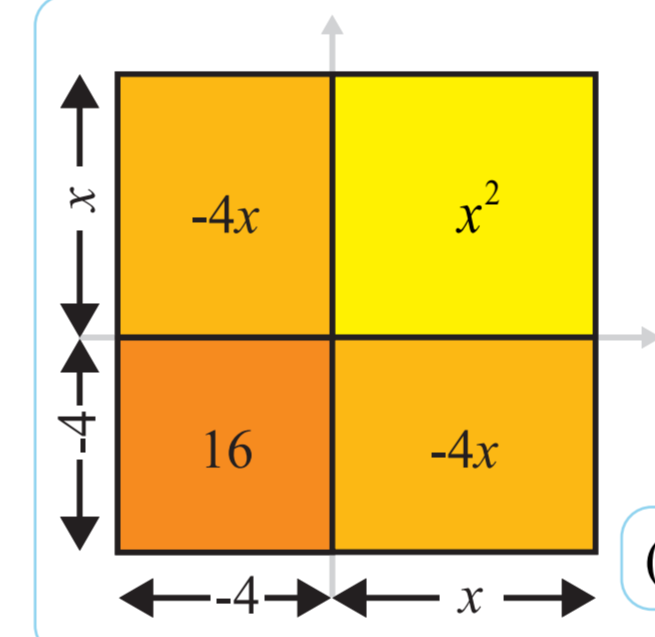


Mae'r gromlin yn cyffwrdd â'r echelin x yn -3. Allwch chi feddwl pam?

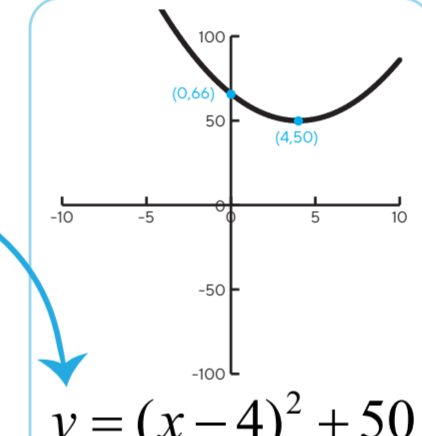
Trawsfudiadau #1

Pan fydd graff ffwythiant cwadratig sgwâr perffaith yn cael ei lunio, mae'n drawsffudiad yn y cyfeiriad x.

$(3x)^2 = 9x^2$



Beth sy'n wahanol? Pam? Beth sydd yr un peth? Pam?



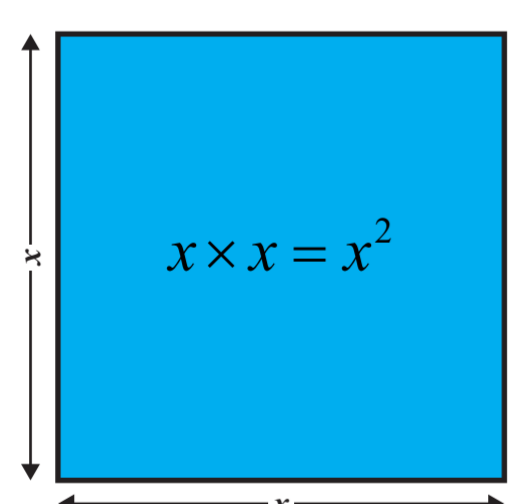
$(x-4)^2 + 1 = x^2 - 8x + 17$

Pan allwn gymryd mynegiad cwadratig a'i ysgrifennu yn y ffurf hon, rydym yn ei alw'n **cwblhau'r sgwâr**.

$(x+p)^2 + q$

Sgwario anhysbysyn

Pan fyddwn yn sgwario rhif, nid ydym ei wybod eto rydym yn defnyddio llythyren i gynrychioli'r rhif hwn. Gallai fod yn unrhyw lythyren o unrhyw wyddor, yn aml mae'n x, diolch i Descartes eto am hyn!



Yn Berffaith Sgwâr

Mae unrhyw beth y gellir ei ysgrifennu fel rhywbeth wedi'i sgwario yn cael ei alw'n **sgwâr perffaith**. Mae'r rhain i gyd yn **sgwariau perffaith**:

$(x+3)^2$ $(3x)^2$ $(x-4)^2$
 7^2 $(x+y+12-5z)^2$

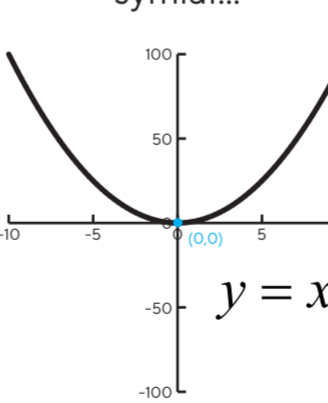
Cwadratigau!

Gelwir mynegiad sydd ag anhysbysion i bŵer uchaf o 2, ac efallai rhai anhysbysion, ac o bosibl cysonyn, yn gwadratig. Ni chaniateir unrhyw bŵerau eraill. Mae'r rhain i gyd yn fynegiadau cwadratig:

$(x+3)^2$ $12-r^2+3r$ x^2+42
 $(t-10)^2-5$ x^2 p^2+p-12

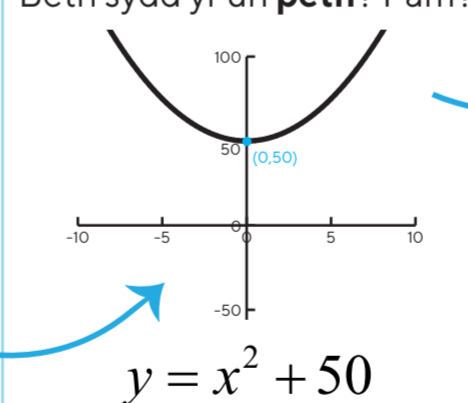
Plotiwch e!

Plotio'r cwadratig symlaf...



Huh?

Beth sy'n wahanol? Pam? Beth sydd yr un peth? Pam?

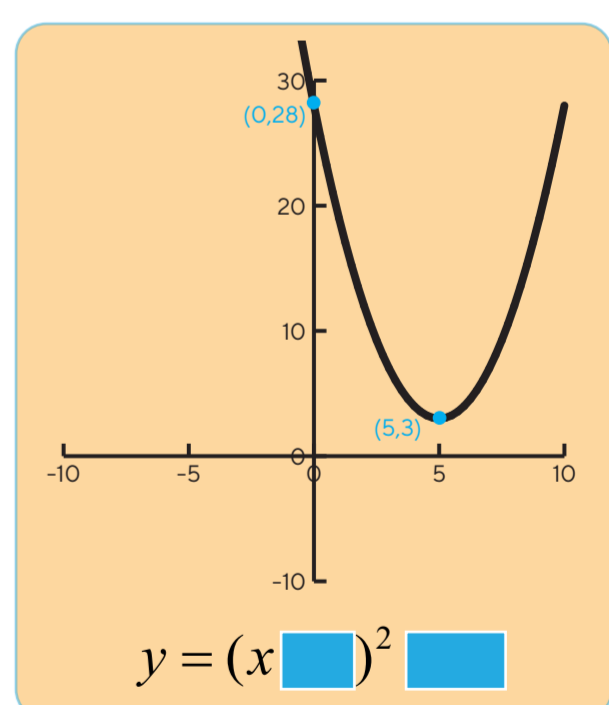


Trawsfudiadau #2

Beth sydd wedi achosi'r trawsffudiad fertigol yn y cyfeiriad y? Wyt ti'n gallu gweld?

Beth yw e?

Trwy edrych ar y graff i'r dde, a allwch chi benderfynu ei hafaliad. Beth sy'n mynd yn y blychau glas isod?



Mae mwy i Fathemateg nag ydych chi'n feddwl.... ymwelwch â rhgmc-mspw.cymru i ddarganfod mwy.

Gellir astudio **Lefel 2 Mathemateg Ychwanegol** yn ystod cyfnod allweddol pedwar.

Yng nghyfnod allweddol pump **Safon Uwch Mathemateg** yw'r **Safon Uwch mwyaf poblogaidd** a **Mathemateg Bellach Safon Uwch** yw'r cydymaith perffaith.

Mae Rhaglen Gymorth Mathemateg Cymru (RhGMC) yma i gefnogi myfyrwyr, athrawon ac adrannau ledled Cymru wrth gyfoethogi a datblygu eu parth Mathemategol ar draws pob cyfnod allweddol.

Cyfoethogi + Dysgu Proffesiynol + Hyfforddiant + Adnoddau + Ymchwil



[youtube.com/c/RhGMC MSPW](https://www.youtube.com/c/RhGMC MSPW)

rhgmc-mspw@swansea.ac.uk

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Funded by Welsh Government