Al-Karaji's Triangle
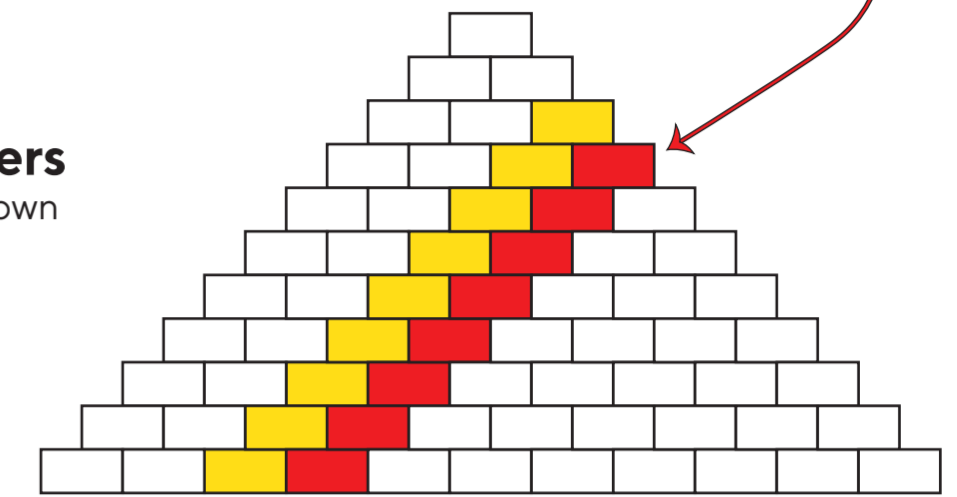
Who knew!
Within this simple triangle lay many, many mathematical wonders.
Here are just a few....

The Persian mathematician **Al-Karaji** wrote a book detailing the first formulation of this triangle in the 10th century. Other mathematicians from different cultures have also written about this triangle, including the Persian mathematician **Omar Khayyam** and Chinese mathematician **Jia Xian** in the 11th century. In Western culture, **Blaise Pascal** was the French Mathematician whom this triangle is often named after. With 1s in all the bricks on the sloping sides, all other bricks are the sum of the two bricks which touch its upper edge. **What value should be in the ? brick?**

The Tetrahedral Numbers
In the red bricks live the tetrahedral numbers.
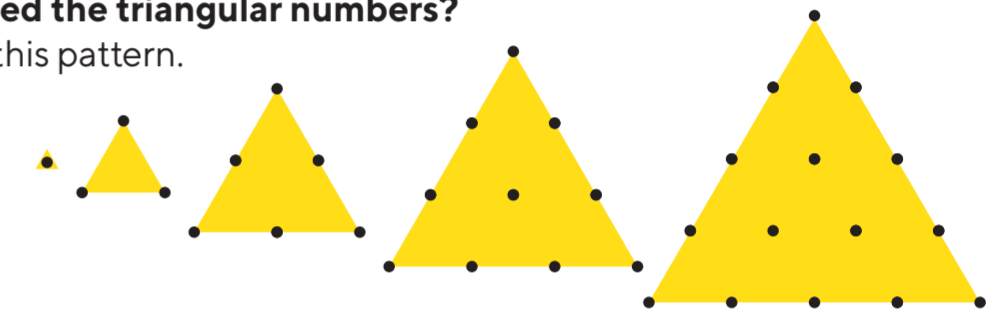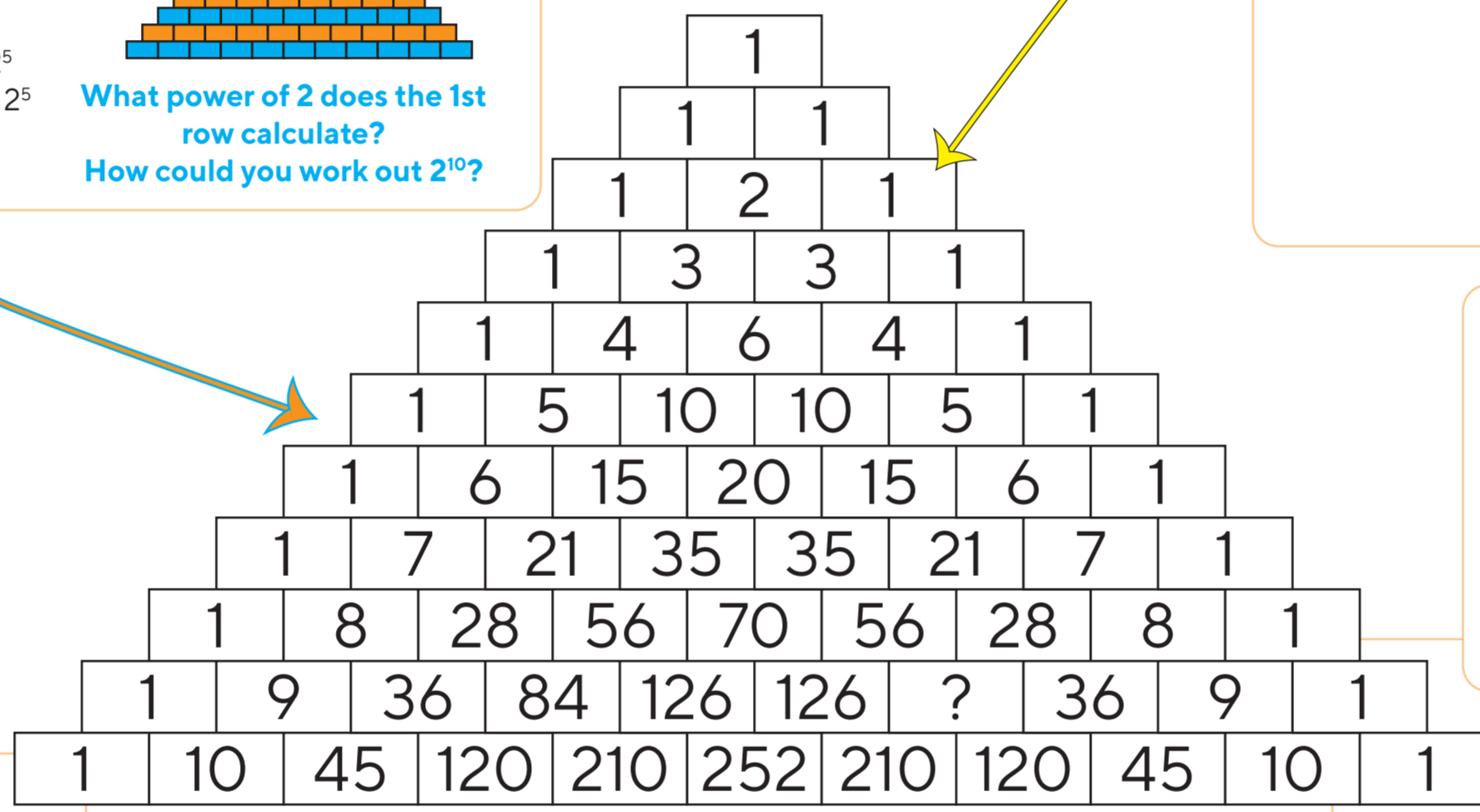**Can you draw the pattern they make?**

The Triangular Numbers
If you follow the yellow bricks down through each layer, it reveals the triangular numbers.

Do you know why they are called the triangular numbers?
Take a look at this pattern.

Hexagons
Calculate the product of the alternating vertices of any hexagon within Al-Karaji's Triangle.

$6 \times 5 \times 20 = ?$
$4 \times 15 \times 10 = ?$

6 4
10 5
20 15

Anything interesting?

The Powers of Two
If you calculate the total of each row a special set of numbers is revealed.

The 6th row calculates 2^5
$1 + 5 + 10 + 10 + 5 + 1 = 32 = 2^5$

What power of 2 does the 1st row calculate?
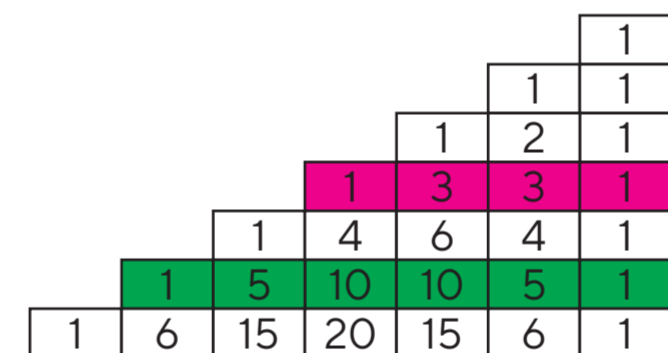**How could you work out 2^{10}?**

Combinatorics
If we have a set of objects and you wish to choose a certain number of them, this triangle tells you how many possible ways there are to do this.
E.g. if you have 4 books on the shelf, I blindly pick any 2, there are 6 different pairs of books I could choose.

		0 objects Choose 0		
	1 object Choose 0		1 object Choose 1	
2 objects Choose 0		2 objects Choose 1		2 objects Choose 2

3 objects Choose 0, 3 objects Choose 1, 3 objects Choose 2, 3 objects Choose 3
4 objects Choose 0, 4 objects Choose 1, 4 objects Choose 2, 4 objects Choose 3, 4 objects Choose 4
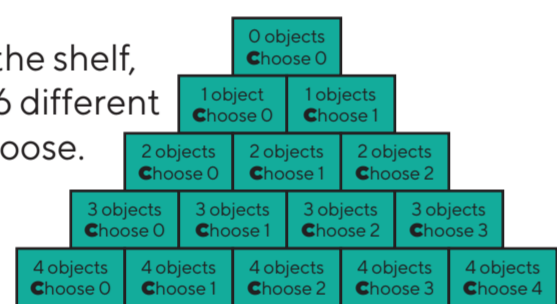
If you have 6 books and wish to choose 3 of them, how many different combinations are there?
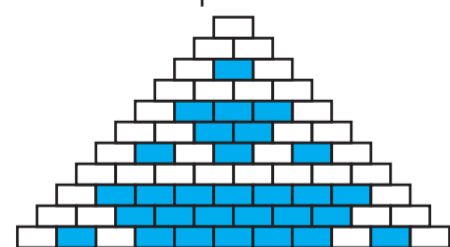
The Powers of Eleven
Right align the bricks in the triangle. Each row gives you a power of 11...
$11^0 = 1$, $11^1 = 11$, $11^2 = 121$ etc,.

**What is 11^3?**
Can you figure out 11^5? Be careful!

10^6	10^5	10^4	10^3	10^2	10^1	10^0
						1
					1	1
				1	2	1
			1	3	3	1
		1	4	6	4	1
	1	5	10	10	5	1
1	6	15	20	15	6	1

The Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 ? 36 9 1
1 10 45 120 210 252 210 120 45 10 1

Multiples
If you shaded all the multiples of 2

If you shaded all the multiples of 3

What patterns do you create with other multiples? Try 4, 5, 6, 7, ...
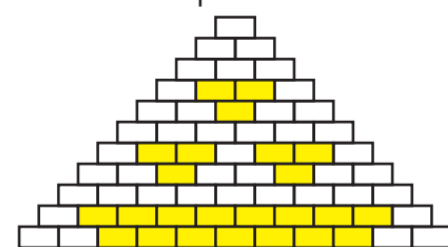
Binomials
An algebraic expression with 2 parts is called a binomial. If raised to different integer (whole number) powers, we can use Al-Karaji's triangle to help us determine the coefficients.

$(x+y)^0 = 1$
$(x+y)^1 = 1x + 1y$
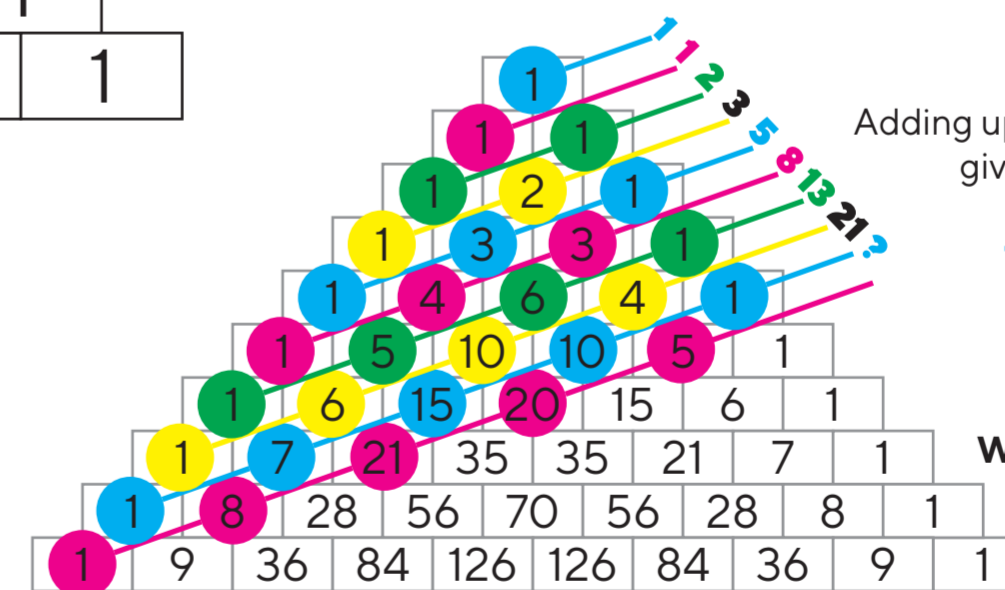$(x+y)^2 = 1x^2 + 2xy + 1y^2$
$(x+y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3$
$(x+y)^4 = 1x^4 + \_\_\_ + \_\_\_ + \_\_\_ + 1y^4$

Can you expand the binomial to the power of 4 using the triangle to help you?

Fibonacci
Adding up along these coloured diagonals gives you the numbers in the Fibonacci sequence
1, 1, 2, 3, 5, 8, 13, 21, ...

What's next? Is there any other way to determine the next number?

αβγδεζηθικλμνξοπρστυφχψω

Trigonometry

The word **trigonometry** has evolved from **trigonon** meaning three (tri) angles (gon) and **metria** meaning to measure.

Imagine
If we take this right angled triangle and enlarge it and shrink it, to lots of different sizes, keeping all the sides in proportion....

What do you notice?
Did you spot all the angles stayed the same.

A Different Angle...?
If we change the size of the original angle in the triangle, and do some more enlargements, again the angle stays the same in all of the new triangles we create.

If Sine, relates to the opposite and hypotenuse sides, how do you think Cos(ine) and Tan(gent) are related?

Give them a name!
Naming the sides, depending on where they are in relation to the angle, helps us to calculate things.

Hypotenuse

Opposite

Adjacent

The opposite side is opposite to the angle in question.

The adjacent side is next to the angle in question.

In each of these three triangles, divide the Opposite by the Hypotenuse.

12cm, 8cm, 7.5cm, 5cm, 15cm, 10cm

$$\frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{8}{12} = \frac{2}{3} \qquad = \frac{5}{7.5} = \frac{2}{3} \qquad = \frac{10}{15} = \frac{2}{3}$$

In each of these three triangles, divide the Adjacent by the Hypotenuse.

12cm, 15cm, 5cm, 4cm, 6cm, 7.5cm

$$\frac{\text{Adjacent}}{\text{Hypotenuse}} = \frac{12}{15} = \frac{4}{5} \qquad = \frac{4}{5} \qquad = \frac{6}{7.5} = \frac{4}{5}$$

So
When the angle stays the same, all the sides stay in the same proportions.

For any given angle
The ratio of any two sides is always the same. This is because when the angle is fixed we know all the triangles are similar, and therefore sides must be in proportion.
When we find the ratio of any two sides for a fixed angle, we will always get the same answer. If we change the angle the ratio will change.

To 360° and beyond
We can continue to increase the angle and move the triangle around the origin, if we continue to plot the height of the triangle the graph below is drawn.

What do you think happens as the angle exceeds 360°?

The Sine Wave
This function is called the Sine function (shortened to **sin**). It carries on forever, as you continue around and around the circle. Luckily we have a button on the calculator which stores this graph and determines it's values when we need them.

sin | cos | tan

Bigger than 90°?
If we draw our triangles in a unit circle, (often, in maths **unit** is a word used for 1, so a **unit circle** has a radius of 1). As the hypotenuse is 1, the height of the triangle represents the ratio of the opposite and hypotenuse. If we define the angle as always being measured from the positive horizontal axis we can move the triangle around the origin.

As the angle gets closer to 90°, the height of the triangle cannot exceed 1.

θ

Let's plot it
If we draw a graph of the different **angles** on the horizontal axis and the **ratio** of the opposite and hypotenuse on the vertical axis, we get:

$\frac{\text{Opposite}}{\text{Hypotenuse}}$

As the angle gets closer to 90°, the ratio seems to get no higher than 1.

What do you think happens to the graph with angles over 90°? Would it still be a right angled triangle?

Angle

There is more to Mathematics than you think.... visit rhgmc-mspw.cymru to find out more.
Level 2 Additional Maths can be studied during key stage four.
In key stage five A Level Mathematics is the most popular[1] A–level and A Level Further Mathematics is the perfect accompaniment.

The Mathematics Support Programme Wales (MSPW) is here to support students, teachers and departments across Wales in enriching and developing their Mathematical domain across all key stages.

Enrichment + Professional Learning + Tuition + Resources + Research

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[1]https://www.wjec.co.uk/media/invh1fni/gce-a-level-provisional-results-june-2022.pdf

RhGMC
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Quadrare

Quadrare
Is a Latin word meaning 'to make square', it's where the words
quadratics, **quad**rangle, **quad**rilateral, **quad**riceps... come from.
Can you think of any more quad words?

Not everything is perfect
Just like in the real world, not everything in maths can be perfect –
although much of maths is pretty good! There are lots of expressions
that cannot be written as a perfect square

Make a square
When we square a number we can show
this in a picture, it makes a square!

$5^2 = 5 \times 5 = 25$
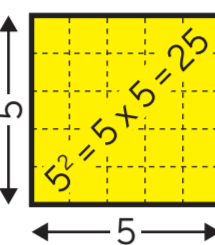
Squaring a number
When we square a number, we multiplying by itself.
5 x 5 = 25 or **9 x 9 = 81** or **–6 x –6 = 36**
We use a superscript 2 to be the symbol for this
operation. René Descartes developed this notation.
5² = 25 or **9² = 81** or **(–6)² = 36**

$(-6)^2 = -6 \times -6 = 36$

$9^2 = 9 \times 9 = 81$

Squaring an expression
Pretty much anything can be squared. And we
can always draw a square diagram to show it.

$(3x)^2$

$(3x)^2 = 9x^2$

$3x \quad 9$
$x^2 \quad 3x$

$(x+3)^2 = x^2 + 6x + 9$
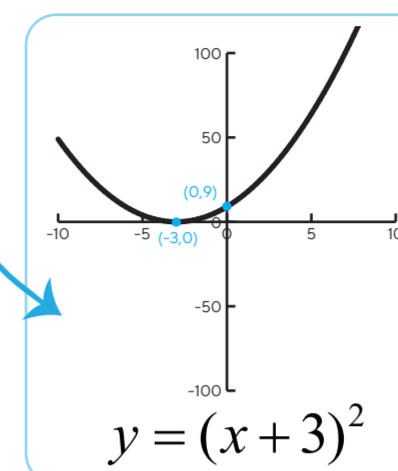
The curve touches
the x axis at –3. Can
you think why?

$y = (x+3)^2$

Completing the square
When an expression can't be written as a
perfect square

$(x-4)^2 \quad = x^2 - 8x + 17$

This expression is almost a **perfect
square**, but we need to add 1 to the left
side of expression for it to be identical.

$(x-4)^2 + 1 = x^2 - 8x + 17$

When we can take a quadratic
expression and write it in this form,
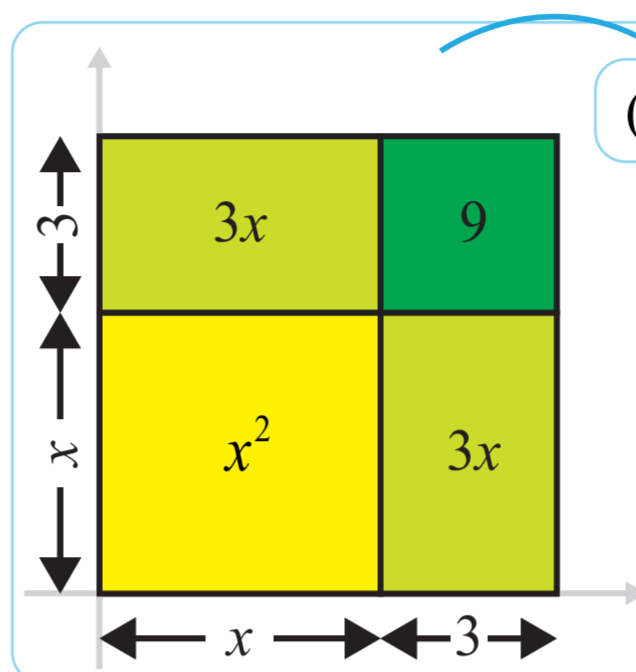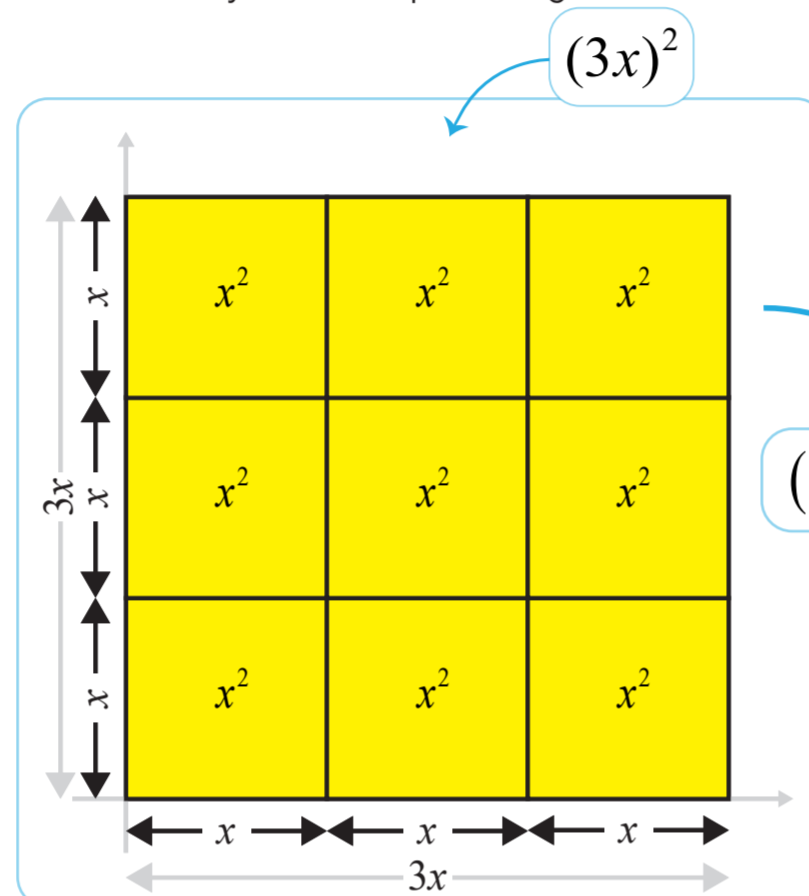we call it **completing the square**.

$(x+p)^2 + q$

Squaring an unknown
When we square a number, we don't know yet
we use a letter to represent this number. It could
be any letter from any alphabet, quite often it's x,
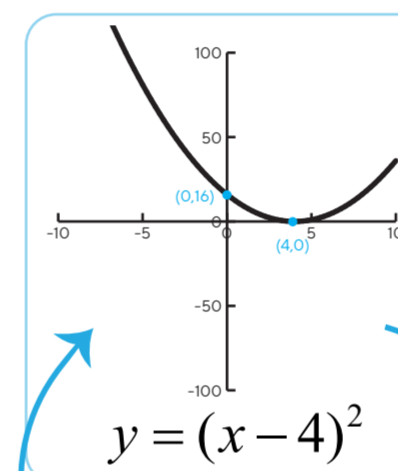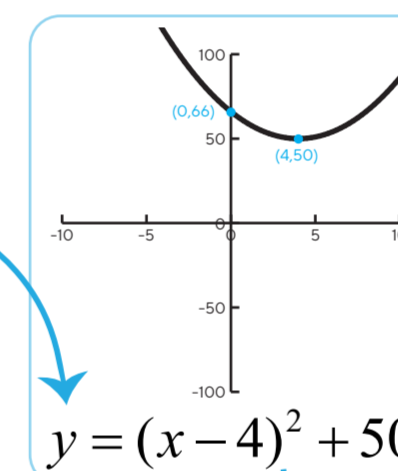thanks to Descartes again for this!

$x \times x = x^2$

Perfectly Square
Anything that can be written as something
squared is called a **perfect square**. All of
these are **perfect squares**:

$(x+3)^2 \quad (3x)^2 \quad (x-4)^2$
$7^2 \quad (x+y+12-5z)^2$

Translations #1
When the graph of a perfect square
quadratic function is drawn, it is a
translation in the x direction.

$-4x \quad x^2$
$16 \quad -4x$

What's **different**? Why?
What's the **same**? Why?

$y = (x-4)^2$

$(x-4)^2 = x^2 - 8x + 16$

$y = (x-4)^2 + 50$

Quadratics!
An expression which has unknowns to a highest power of 2,
and maybe some unknowns and possibly a constant,
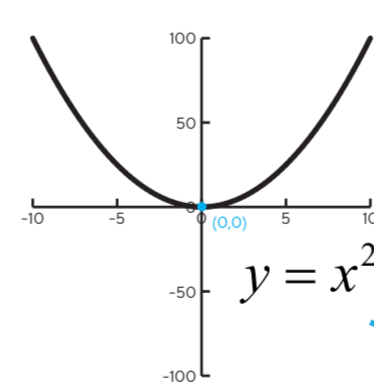is called a Quadratic. No other powers allowed.
These are all quadratic expressions:

$(x+3)^2 \qquad 12 - r^2 + 3r \qquad x^2 + 42$
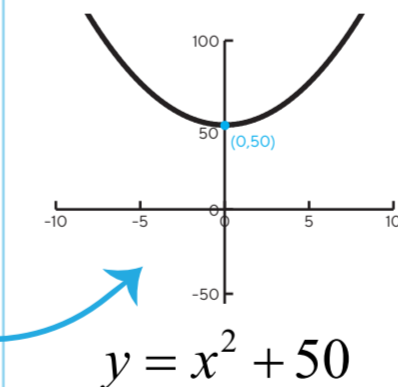
$(t-10)^2 - 5 \qquad x^2 \qquad p^2 + p - 12$

Plot it!
Plotting the simplest
quadratic...

$y = x^2$

Huh?
What's **different**? Why?
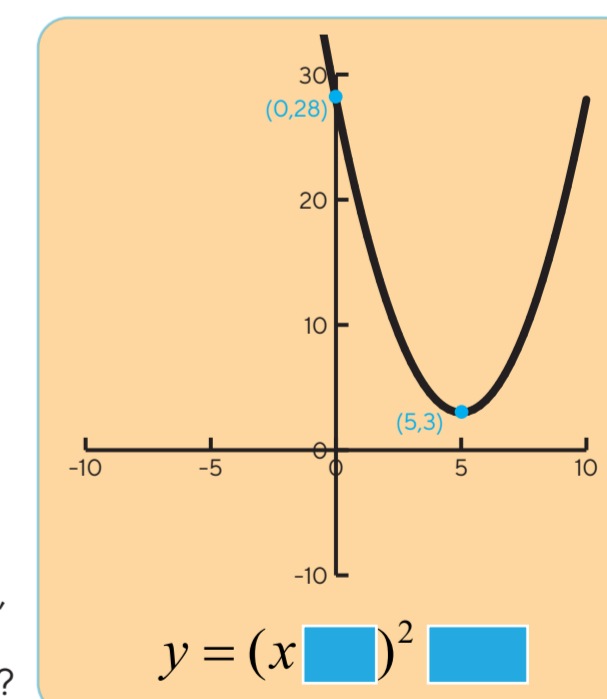What's the **same**? Why?

$y = x^2 + 50$

Translations #2
What has caused the
vertical translation in the
y direction? Can you see?

What is it?
By looking at the graph to the right,
can you determine it's equation.
What goes in the blue boxes below?

$y = (x \; \Box)^2 \; \Box$

Answer: $y = (x-5)^2 + 3$

Triongl Al-Karaji

Pwy wyddai!
O fewn y triongl syml hwn roedd llawer o ryfeddodau mathemategol.
Dyma rai yn unig...

Ysgrifennodd y mathemategydd Persiaidd **Al-Karaji** lyfr yn manylu ar fformwleiddiad cyntaf y triongl hwn yn y 10fed ganrif. Mae mathemategwyr eraill o wahanol ddiwylliannau hefyd wedi ysgrifennu am y triongl hwn, gan gynnwys y mathemategydd Persiaidd **Omar Khayyam** a'r mathemategydd Tsieineaidd **Jia Xian** yn yr 11fed ganrif. Yn niwylliant y Gorllewin, **Blaise Pascal** oedd y mathemategydd Ffrengig y mae'r triongl hwn yn aml yn cael ei enwi ar ei ôl.   Gyda 1 yn yr holl frics ar yr ochrau sy'n goleddu, mae pob un o'r brics eraill yn swm y ddwy fricsen sy'n cyffwrdd a'i ymyl uchaf. **Pa werth ddylai fod yn y fricsen?**
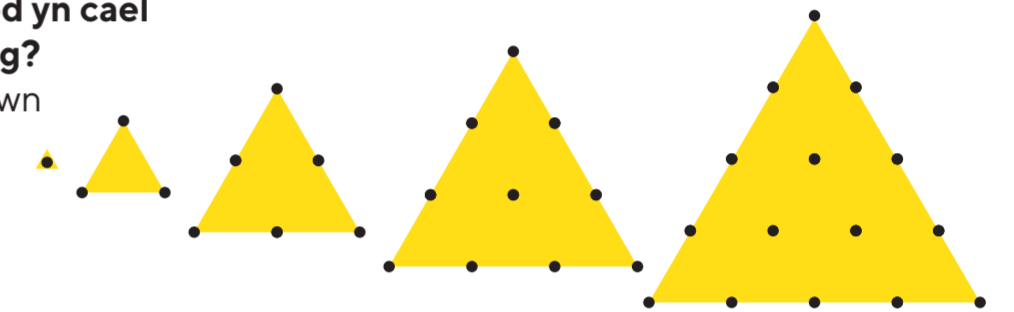
Y Rhifau Tetrahedrol
Yn y brics coch y mae'r rhifau tetrahedrol yn byw.
Allwch chi lunio'r patrwm maen nhw'n ei wneud?
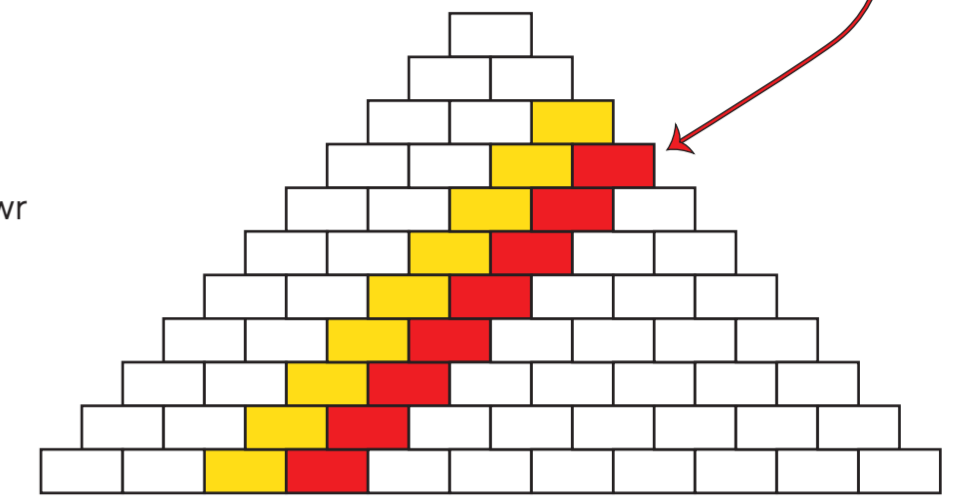
Y Rhifau Triongl
Os dilynwch y brics melyn i lawr
trwy bob haen,
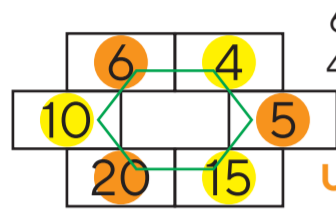Mae'n datgelu'r
rhifau trionglog.

Ydych chi'n gwybod pam eu bod yn cael eu galw'n rhifau trionglog?
Edrychwch ar y patrwm hwn

Hecsagonau
Cyfrifwch luoswm fertigau eiledol unrhyw hecsagon o fewn Triongl Al-Karaji.

6 x 5 x 20 = ?
4 x 15 x 10 = ?

Unrhyw beth diddorol?

Pwerau dau
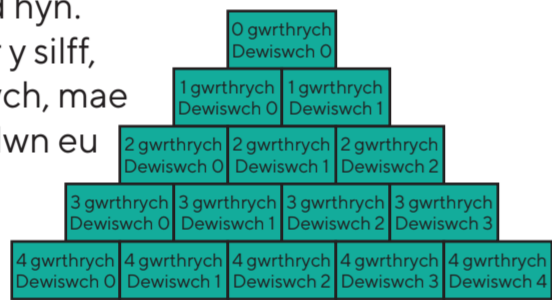Os ydych chi'n cyfrifo cyfanswm pob rhes mae set arbennig o rifau yn cael ei ddatgelu.

Mae'r 6ed rhes yn cyfrif 2^5
$1 + 5 + 10 + 10 + 5 + 1 = 32 = 2^5$

Pa bŵer o 2 mae'r rhes 1af yn ei gyfrifo?
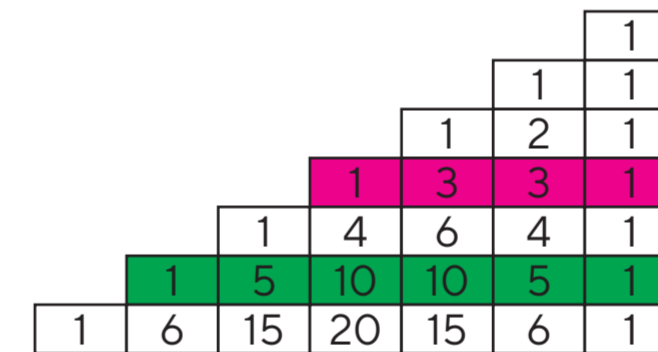**Sut allwch chi gyrraedd 2^{10}?**
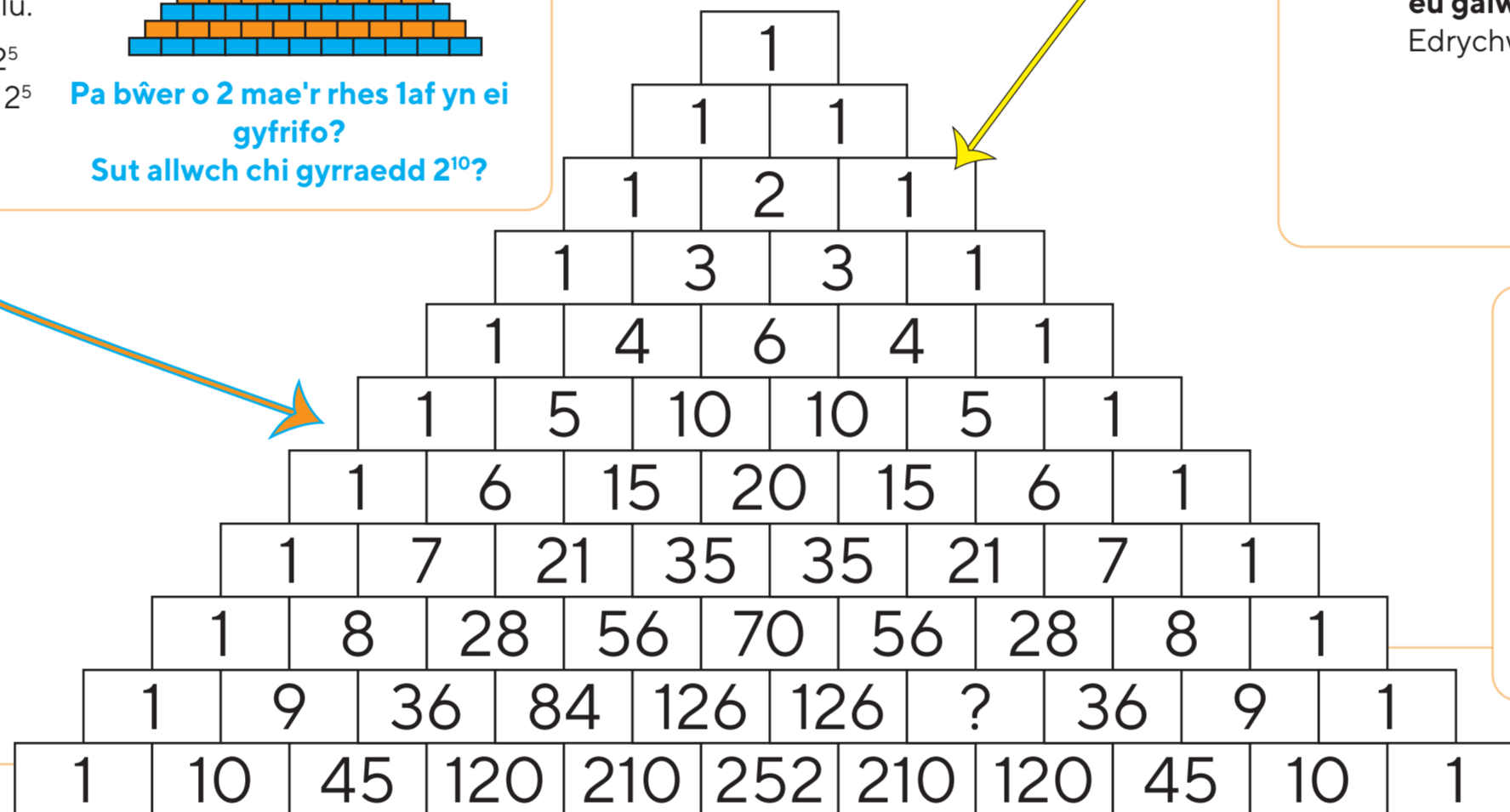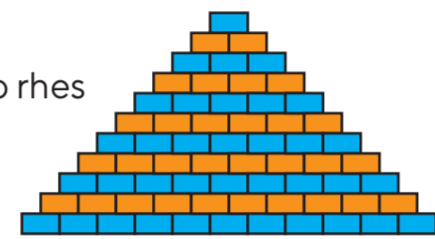
Cyfuniadeg
Os oes gennym set o wrthrychau a'ch bod yn dymuno dewis rhai ohonynt, mae'r triongl hyn yn dweud wrthych faint o ffyrdd posibl sydd i wneud hyn.
E.e. Os oes gennych 4 llyfr ar y silff, rwy'n dewis unrhyw 2 heb edrych, mae 6 pâr gwahanol o lyfrau y gallwn eu dewis.

0 gwrthrych Dewiswch 0
1 gwrthrych Dewiswch 0 / 1 gwrthrych Dewiswch 1
2 gwrthrych Dewiswch 0 / 2 gwrthrych Dewiswch 1 / 2 gwrthrych Dewiswch 2
3 gwrthrych Dewiswch 0 / 3 gwrthrych Dewiswch 1 / 3 gwrthrych Dewiswch 2 / 3 gwrthrych Dewiswch 3
4 gwrthrych Dewiswch 0 / 4 gwrthrych Dewiswch 1 / 4 gwrthrych Dewiswch 2 / 4 gwrthrych Dewiswch 3 / 4 gwrthrych Dewiswch 4

Os oes gennych 6 llyfr ac yn dymuno dewis 3 ohonynt, faint o gyfuniadau gwahanol sydd yna?

Lluosrifau
Os ydych chi'n tywyllu holl luosrifau 2
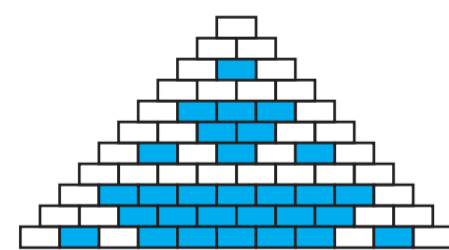
Os ydych chi'n tywyllu holl luosrifau 3
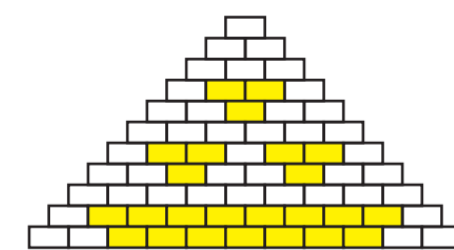
Pa batrymau ydych chi'n eu creu gyda lluosrifau eraill? Rhowch gynnig ar 4, 5, 6, 7, ...

Binomialau
Gelwir mynegiad algebraidd gyda 2 ran yn finomial. Os caiff ei godi i wahanol bwerau cyfanrif, gallwn ddefnyddio triongl Al-Karaji i'n helpu i bennu'r cyfernodau.

$$(x+y)^0 = 1$$
$$(x+y)^1 = 1x + 1y$$
$$(x+y)^2 = 1x^2 + 2xy + 1y^2$$
$$(x+y)^3 = 1x^3 + 3x^2y + 3xy^2 + 1y^3$$
$$(x+y)^4 = 1x^4 + \underline{} + \underline{} + \underline{} + 1y^4$$

Allwch chi ehangu'r binomial i bŵer 4 gan ddefnyddio'r triongl i'ch helpu?

Triongl (Pascal)

					1					
				1		1				
			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1

1, 6, 15, 20, 15, 6, 1
1, 7, 21, 35, 35, 21, 7, 1
1, 8, 28, 56, 70, 56, 28, 8, 1
1, 9, 36, 84, 126, 126, ?, 36, 9, 1
1, 10, 45, 120, 210, 252, 210, 120, 45, 10, 1

Pwerau 11
Aliniwch y brics i'r dde yn y triongl.
Mae pob rhes yn rhoi pŵer i chi o 11...
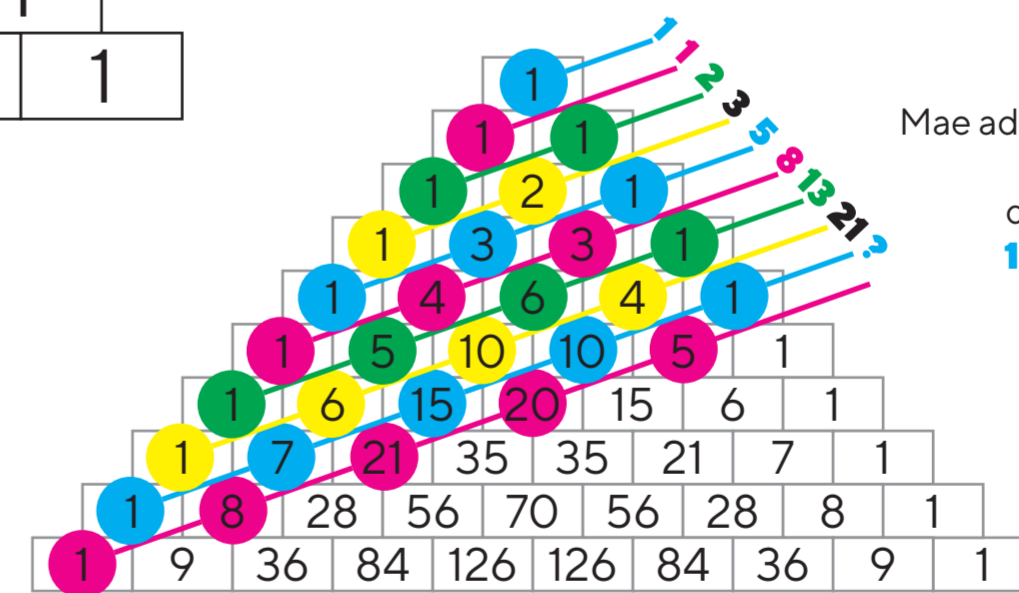$11^0 = 1$, $11^1 = 11$, $11^2 = 121$ etc,.

10^6	10^5	10^4	10^3	10^2	10^1	10^0
						1
					1	1
				1	2	1
			1	3	3	1
		1	4	6	4	1
	1	5	10	10	5	1
1	6	15	20	15	6	1

Beth yw 11^3?
Allwch chi gyfrif 11^5? Bydd yn ofalus!

Fibonacci
Mae adio ar hyd y croesliniau lliw hyn rhoi'r rhifau yn y dilyniant Fibonacci i chi
1, 1, 2, 3, 5, 8, 13, 21, ...

Beth sydd nesaf?
A oes unrhyw ffordd arall o benderfynu ar y rhif nesaf?

Mae mwy i Fathemateg nag ydych chi'n feddwl.... ymwelwch â rhgmc-mspw.cymru i ddarganfod mwy.
Gellir astudio Lefel 2 Mathemateg Ychwanegol yn ystod cyfnod allweddol pedwar.
Yng nghyfnod allweddol pump Safon Uwch Mathemateg yw'r Safon Uwch mwyaf poblogaidd[1] a Mathemateg Bellach Safon Uwch yw'r cydymaith perffaith.

Mae Rhaglen Gymorth Mathemateg Cymru (RhGMC) yma i gefnogi myfyrwyr, athrawon ac adrannau ledled Cymru wrth gyfoethogi a datblygu eu parth Mathemategol ar draws pob cyfnod allweddol.

Cyfoethogi + Dysgu Proffesiynol + Hyfforddiant + Adnoddau + Ymchwil

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α β γ δ ε ζ η θ ι κ λ μ ν ξ ο π ρ σ τ υ φ χ ψ ω
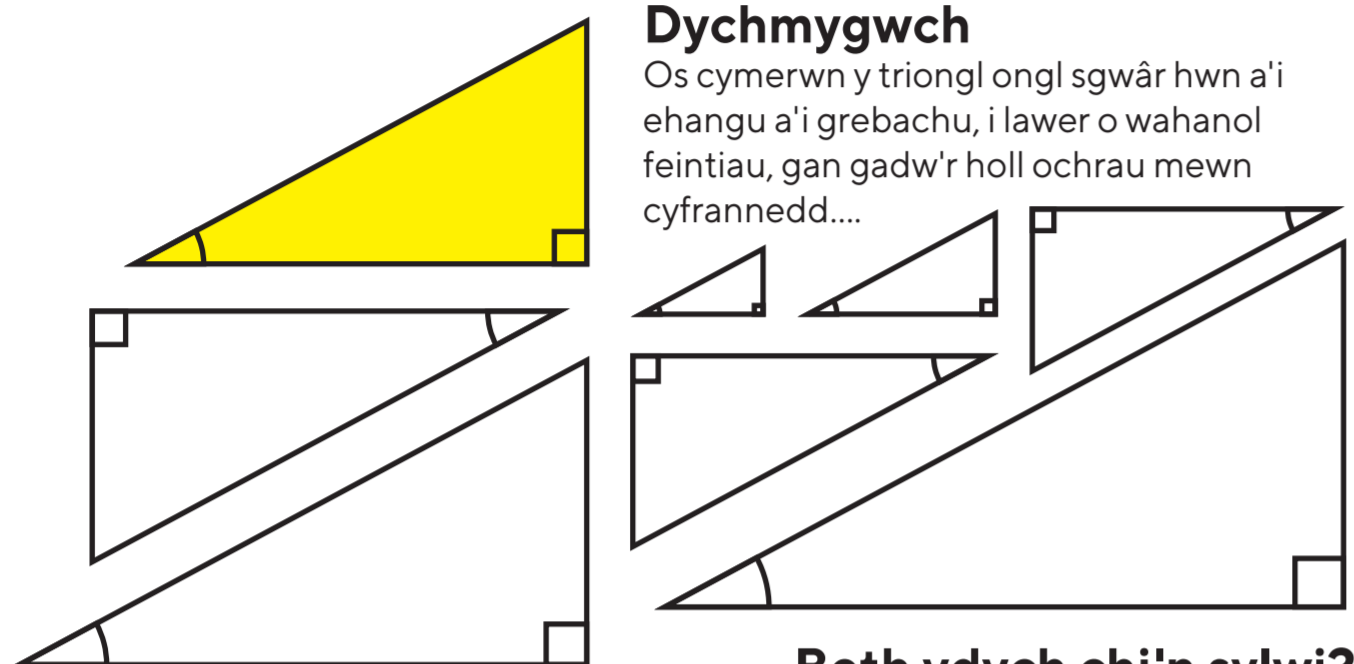
Trigonometreg

Mae'r gair **trigonometreg** wedi esblygu o **trigonon** sy'n golygu tair (tri) ongl (gon) a **metria** sy'n golygu mesur.

Dychmygwch

Os cymerwn y triongl ongl sgwâr hwn a'i ehangu a'i grebachu, i lawer o wahanol feintiau, gan gadw'r holl ochrau mewn cyfrannedd....
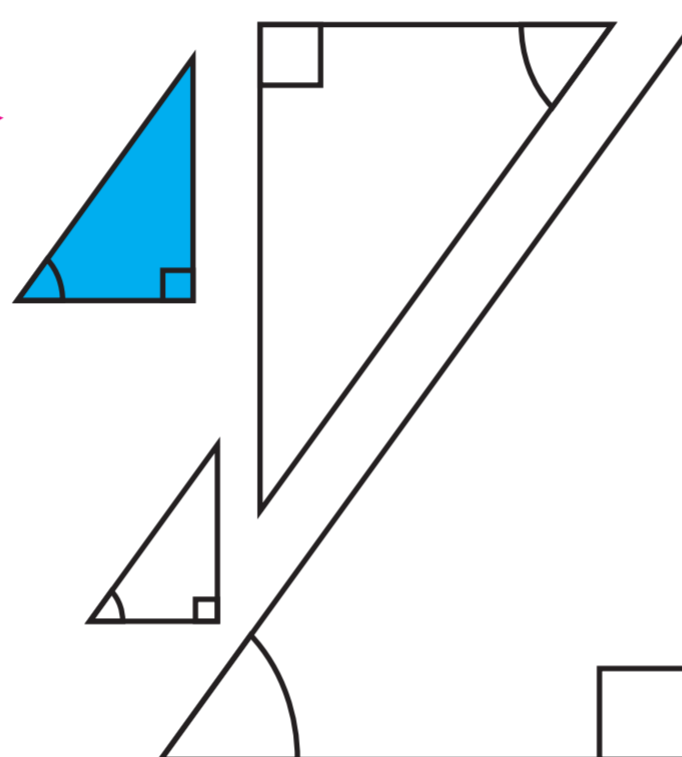
Beth ydych chi'n sylwi?

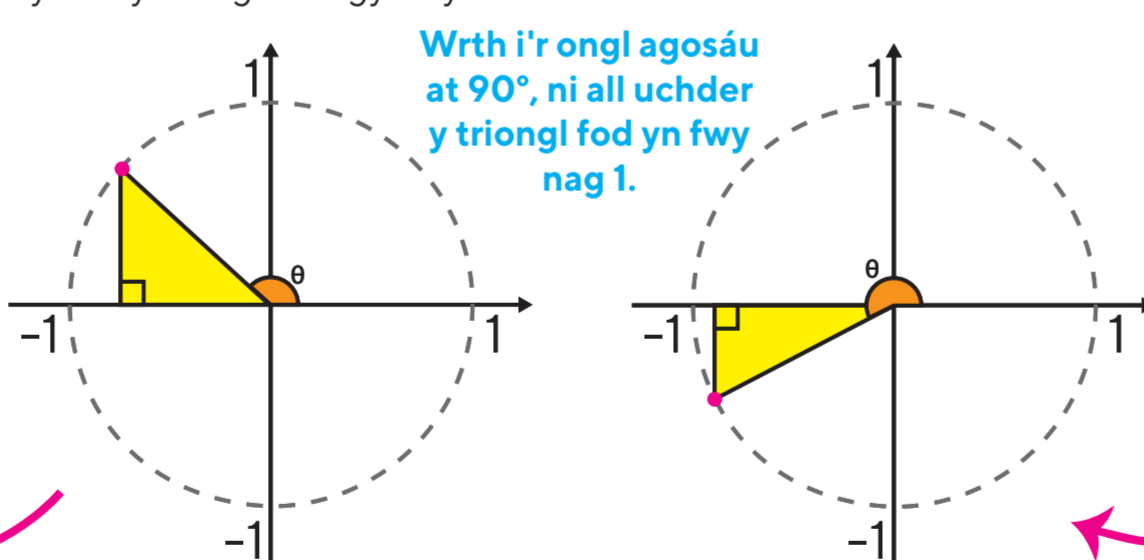Oeddech chi'n sylwi bod yr holl onglau wedi aros yr un fath?

Ongl wahanol...?

Os ydym yn newid maint yr ongl wreiddiol yn y triongl, ac yn gwneud mwy o ehangiadau, eto mae'r ongl yn aros yr un fath ym mhob un o'r trionglau newydd rydyn ni'n eu creu.

Os yw Sin, yn ymwneud â'r ochrau cyferbyn a hypotenws, sut ydych chi'n meddwl bod Cos(in) a Tan(giad) yn gysylltiedig?

Rhowch enw iddyn nhw!

Mae enwi'r ochrau, yn ddibynnol ar ble maen nhw mewn perthynas â'r ongl, yn ein helpu ni i gyfrifo pethau.

Hypotenws
Cyferbyn
Agos

Mae'r ochr gyferbyn yn ddirgroes i'r ongl dan sylw.
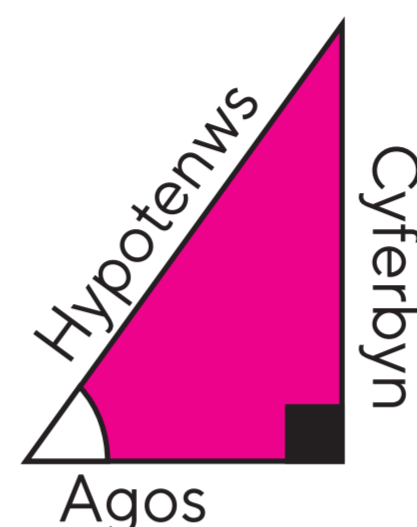
Mae'r ochr agos nesaf i'r ongl dan sylw.

Ym mhob un o'r tri thriongl hyn, rhannwch y Cyferbyn â'r Hypotenws.

12cm, 8cm, 7.5cm, 5cm, 15cm, 10cm
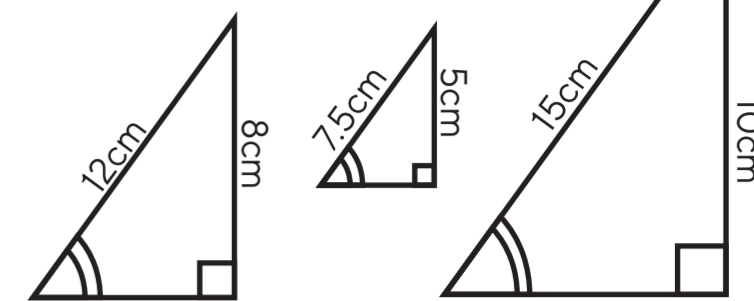
$$\frac{\text{Cyferbyn}}{\text{Hypotenws}} = \frac{8}{12} = \frac{2}{3} \qquad = \frac{5}{7.5} = \frac{2}{3} \qquad = \frac{10}{15} = \frac{2}{3}$$

Ym mhob un o'r tri thriongl hyn, rhannwch yr Agos â'r Hypotenws.

12cm, 15cm, 5cm, 4cm, 6cm, 7.5cm
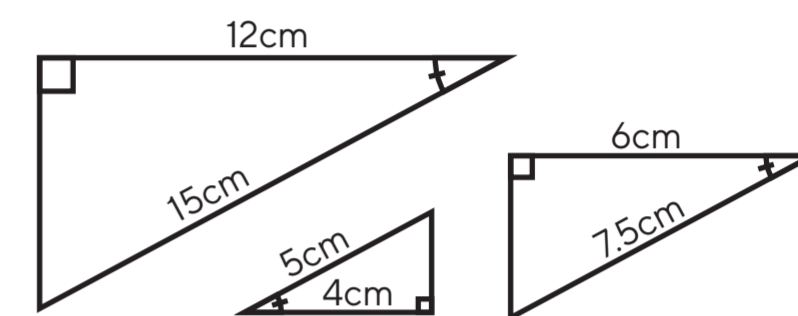
$$\frac{\text{Agos}}{\text{Hypotenws}} = \frac{12}{15} = \frac{4}{5} \qquad = \frac{4}{5} \qquad = \frac{6}{7.5} = \frac{4}{5}$$

Felly

Pan fydd yr ongl yn aros yr un fath, mae'r holl ochrau'n aros yn yr un cyfrannedd.

360 ° a thu hwnt

Gallwn barhau i gynyddu'r ongl a symud y triongl o amgylch y tardd, os ydym yn parhau i blotio uchder y triongl y mae'r graff isod yn cael ei lunio.
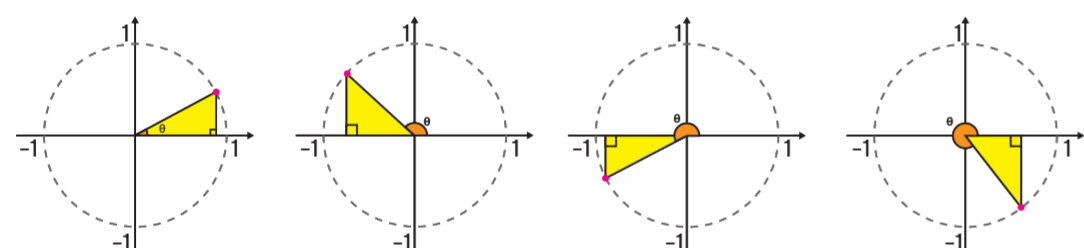
Beth ydych chi'n meddwl sy'n digwydd wrth i'r ongl fod yn fwy na 360°?

Y Don Sin

Gelwir y ffwythiant hwn yn ffwythiant Sin. Mae'n mynd ymlaen am byth, wrth i chi barhau o gwmpas ac o amgylch y cylch. Yn ffodus, mae gennym fotwm ar y cyfrifiannell sy'n storio'r graff hwn ac yn penderfynu ei werthoedd pan fydd eu hangen arnom.

sin | cos | tan

Yn fwy na 90 °?

Os ydym yn tynnu ein trionglau mewn cylch uned, (yn aml, ym mathemateg mae **uned** yn air a ddefnyddir ar gyfer 1, felly mae gan **gylch uned** radiws o 1). Gan fod yr hypotenws yn 1, mae uchder y triongl yn cynrychioli cymhareb y cyferbyn a'r hypotenws. Os ydym yn diffinio'r ongl fel bob amser yn cael ei mesur o'r echelin llorweddol positif, gallwn symud y triongl o amgylch y tardd.

Wrth i'r ongl agosáu at 90°, ni all uchder y triongl fod yn fwy nag 1.

θ θ

Ar gyfer unrhyw ongl benodol

Mae'r gymhareb o unrhyw ddwy ochr bob amser yr un fath. Mae hyn oherwydd pan fydd yr ongl yn sefydlog rydym yn gwybod bod yr holl drionglau yn gyflun, ac felly mae'n rhaid i'r ochrau fod mewn cyfrannedd.
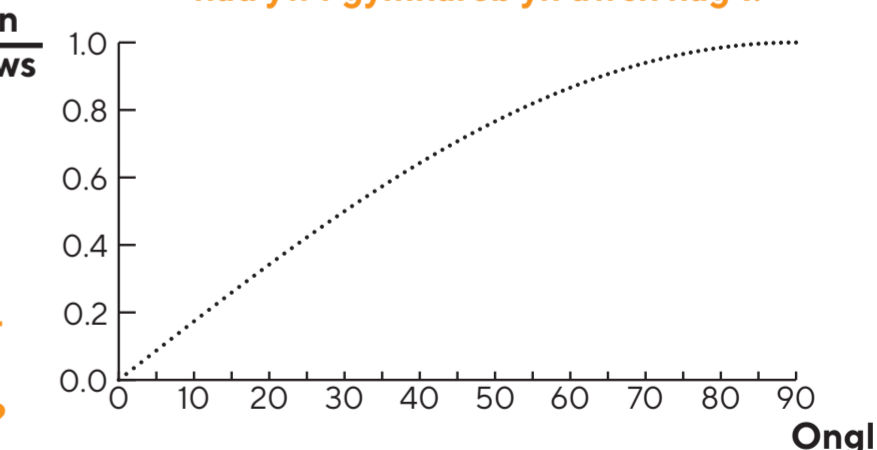
Pan fyddwn yn darganfod cymhareb unrhyw ddwy ochr ar gyfer ongl sefydlog, byddwn bob amser yn cael yr un ateb. Os byddwn yn newid yr ongl bydd y gymhareb yn newid.

Gadewch i ni ei blotio

Os tynnwn graff o'r **onglau** gwahanol ar yr echelin llorweddol a **chymhareb** y cyferbyn a'r hypotenws ar yr echelin fertigol, fe gawn:

$$\frac{\text{Cyferbyn}}{\text{Hypotenws}}$$

Wrth i'r ongl agosáu at 90 °, ymddengys nad yw'r gymhareb yn uwch nag 1.

Beth ydych chi'n meddwl sy'n digwydd i'r graff gydag onglau dros 90°? A fyddai'n dal i fod yn driongl ongl sgwâr?

Ongl

Mae mwy i Fathemateg nag ydych chi'n feddwl.... ymwelwch â rhgmc-mspw.cymru i ddarganfod mwy.
Gellir astudio Lefel 2 Mathemateg Ychwanegol yn ystod cyfnod allweddol pedwar.
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Cyfoethogi + Dysgu Proffesiynol + Hyfforddiant + Adnoddau + Ymchwil

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[1]https://www.wjec.co.uk/media/invh1fni/gce-a-level-provisional-results-june-2022.pdf

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Quadrare

Quadrare

Mae'n air Lladin sy'n golygu 'i wneud yn sgwâr', dyma ble mae'r geiriau **cwad**ratig, **cwad**riceps... yn dod o.
Allwch chi feddwl am fwy o eiriau cwad?

Nid yw popeth yn berffaith.

Yn union fel yn y byd go iawn, ni all popeth ym mathemateg fod yn berffaith - er bod llawer o fathemateg yn eithaf da! Mae yna lawer o fynegiadau na ellir eu hysgrifennu fel sgwâr perffaith

Llunio sgwâr

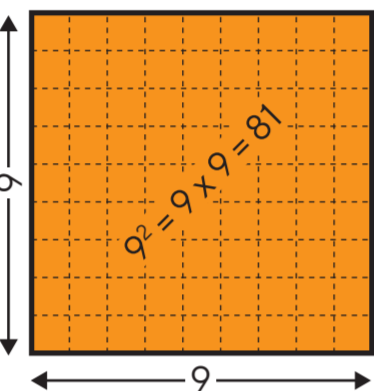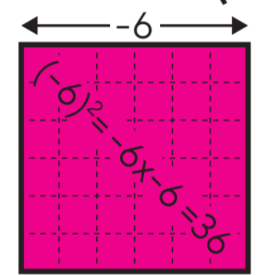Pan fyddwn yn sgwario rhif gallwn ddangos hyn mewn llun, mae'n gwneud sgwâr!

$5^2 = 5 \times 5 = 25$
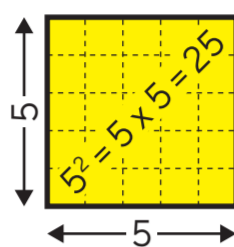
Sgwario rhif

Pan fyddwn yn sgwario rhif, rydym yn ei luosi ag ef ei hun.
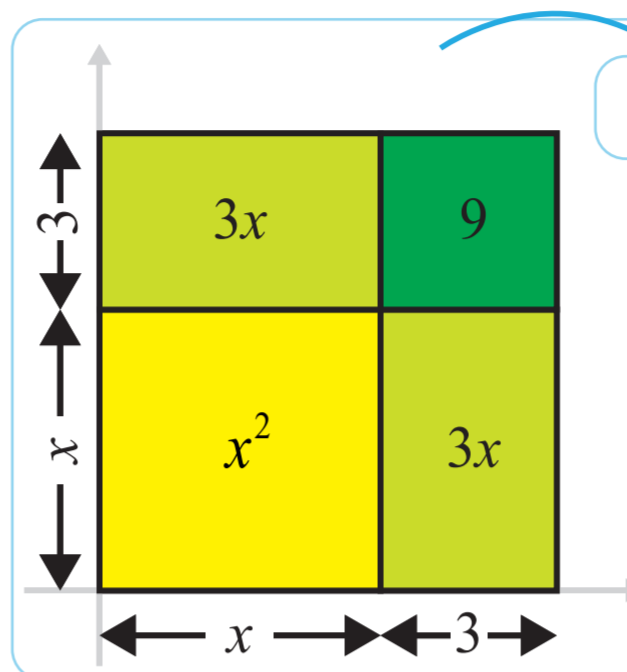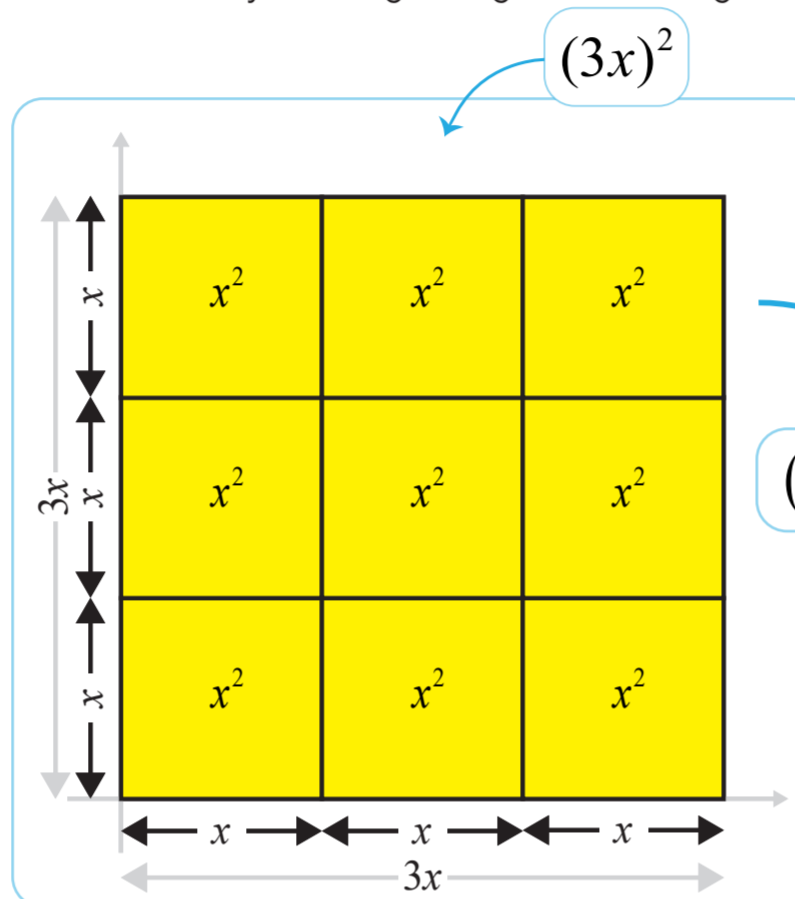**5 x 5 = 25** or **9 x 9 = 81** or **–6 x –6 = 36**
Rydym yn defnyddio uwchysgrif 2 i fod yn symbol ar gyfer y gweithrediad hwn. Datblygodd René Descartes y nodiant hwn.
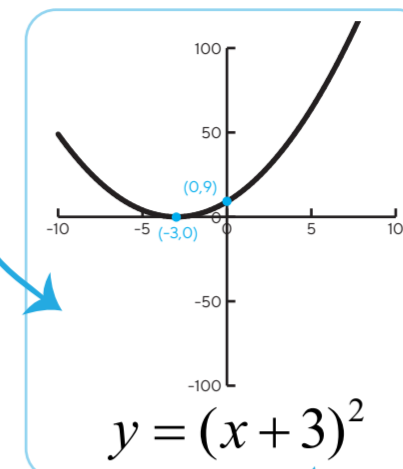**5² = 25** neu **9² = 81** neu **(–6)² = 36**

$(-6)^2 = -6 \times -6 = 36$

$9^2 = 9 \times 9 = 81$

Sgwario mynegiad

Gall bron iawn unrhyw beth gael ei sgwario. A gallwn bob amser dynnu diagram sgwâr i'w ddangos.
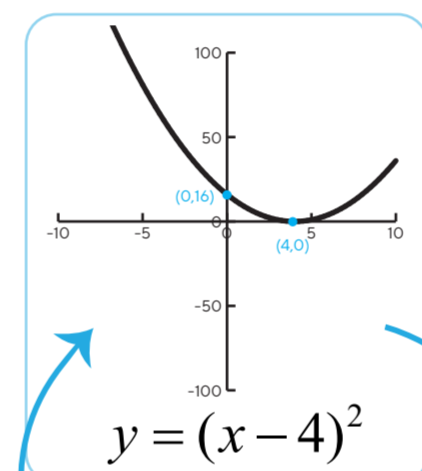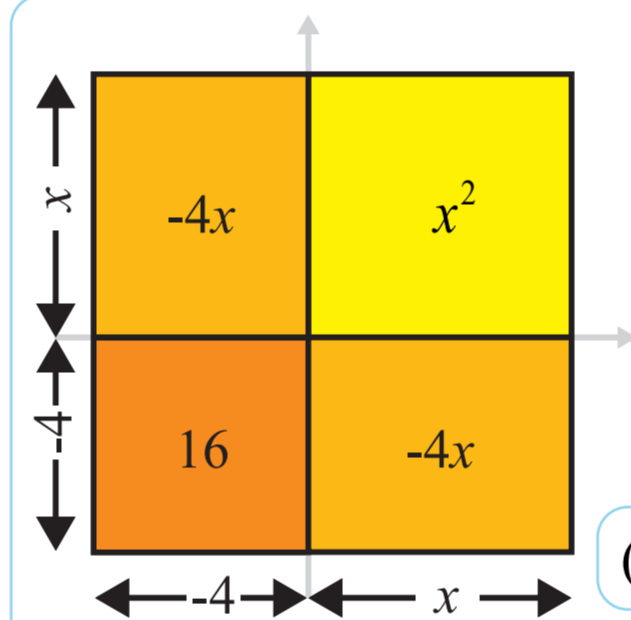
$(3x)^2$

$(3x)^2 = 9x^2$

$(x+3)^2 = x^2 + 6x + 9$

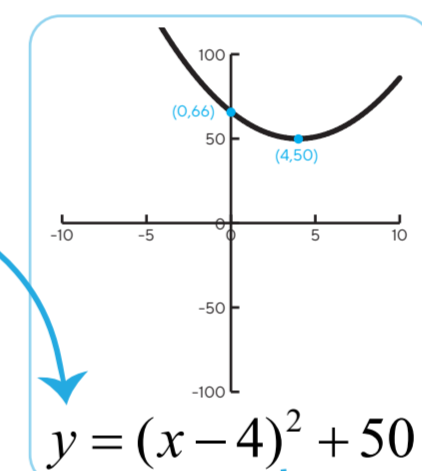Mae'r gromlin yn cyffwrdd â'r echelin x yn –3. Allwch chi feddwl pam?

$y = (x+3)^2$

Trawsfudiadau #1

Pan fydd graff ffwythiant cwadratig sgwâr perffaith yn cael ei lunio, mae'n drawsfudiad yn y cyfeiriad **x**.

Cwblhau'r sgwâr

Pan na ellir ysgrifennu ymadrodd fel sgwâr perffaith

$(x-4)^2 \;\blacksquare\; = x^2 - 8x + 17$

Mae'r ymadrodd hwn bron yn **sgwâr perffaith**, ond mae angen i ni adio 1 at ochr chwith mynegiad er mwyn iddo fod yn unfath.

$(x-4)^2 + 1 = x^2 - 8x + 17$

Pan allwn gymryd mynegiad cwadratig a'i ysgrifennu yn y ffurf hon, rydym yn ei alw'n **cwblhau'r sgwâr**.

$(x+p)^2 + q$

Sgwario anhysbysyn

Pan fyddwn yn sgwario rhif, nid ydym ei wybod eto rydym yn defnyddio llythyren i gynrychioli'r rhif hwn. Gallai fod yn unrhyw lythyren o unrhyw wyddor, yn aml mae'n x, diolch i Descartes eto am hyn!

$x \times x = x^2$

Yn Berffaith Sgwâr

Mae unrhyw beth y gellir ei ysgrifennu fel rhywbeth wedi'i sgwario yn cael ei alw'n **sgwâr perffaith**. Mae'r rhain i gyd yn **sgwariau perffaith**:

$(x+3)^2 \quad (3x)^2 \quad (x-4)^2$
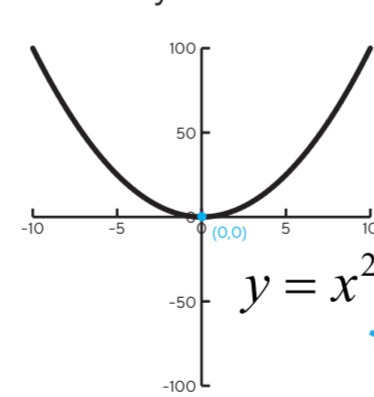
$7^2 \quad (x+y+12-5z)^2$

Cwadratigau!

Gelwir mynegiad sydd ag anhysbysion i bŵer uchaf o 2, ac efallai rhai anhysbysion, ac o bosibl cysonyn, yn gwadratig. Ni chaniateir unrhyw bwerau eraill. Mae'r rhain i gyd yn fynegiadau cwadratig:

$(x+3)^2 \qquad 12 - r^2 + 3r \qquad x^2 + 42$
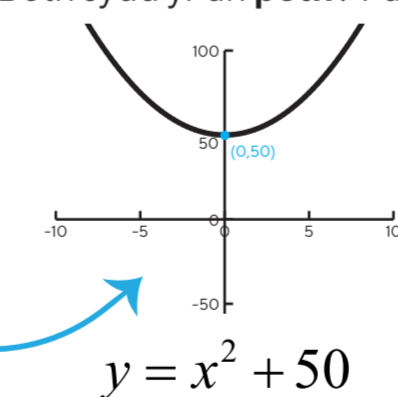
$(t-10)^2 - 5 \qquad x^2 \qquad p^2 + p - 12$

$(x-4)^2 = x^2 - 8x + 16$

$y = (x-4)^2$

Beth sy'n **wahanol**? Pam? Beth sydd yr un **peth**? Pam?

$y = (x-4)^2 + 50$

Trawsfudiadau #2

Beth sydd wedi achosi'r trawsfudiad fertigol yn y cyfeiriad y? Wyt ti'n gallu gweld?

Plotiwch e!

Plotio'r cwadratig symlaf...

$y = x^2$

Huh?

Beth sy'n **wahanol**? Pam? Beth sydd yr un **peth**? Pam?

$y = x^2 + 50$

Beth yw e?

Trwy edrych ar y graff i'r dde, a allwch chi benderfynu ei hafaliad. Beth sy'n mynd yn y blychau glas isod?

$y = (x \;\blacksquare\;)^2 \;\blacksquare\;$
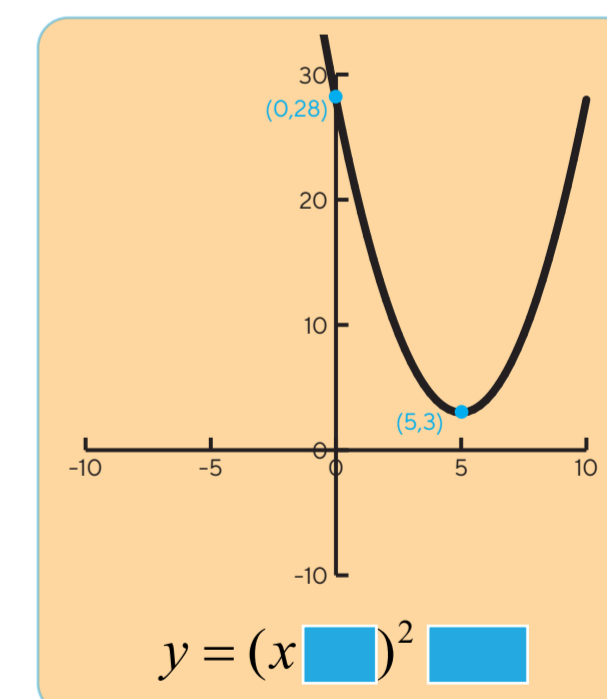
Ateb: $y = (x-5)^2 + 3$

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[1]https://www.wjec.co.uk/media/invh1fni/gce-a-level-provisional-results-june-2022.pdf

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