Informal Proceedings

Connected A Level Programmes of Study

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Introduction

Programmes of Study (PoS) tend to work through specifications topic by topic. Can we improve on this to develop mathematical thinking by looking at the connections in the material and travelling through the mathematics in a different way? The Further Mathematics Support Programme (FMSP) Wales has mapped prior and dependent topics for every topic in our Scheme of Work (SoW) for the new Mathematics (M) and Further Mathematics (FM) A level syllabi. Can we use this resource to grow our students' (and teachers') understanding of the patterns running through mathematics?

FMSP Wales Scheme of Work Connections

Mathematical connection is embedded into our SoW. We have highlighted this through the connections afforded by dependency between topics (see figure 2) and through our requirement for each topic to include:

- bigger picture (history of mathematics, connections to other topics, ideas underpinning mathematical concepts that may go beyond the syllabus, beauty, etc.)
- 'real-world' and workplace applications and connections
- how the topics are reflected in STEM (and subjects using statistics) undergraduate studies
- questions drawing on other topics

Key references include:

- avoiding "…rapid, superficial coverage of a large number of topics each year." (Englemann et al., 2001: 292)
- "maintaining close connections between tasks and children's mathematical thinking" (Clements and Sarama, 2004: 82)
- procedural fluency, relational understanding and schematic understanding, so the learner has a sense of how mathematical actions, processes and objects are connected. Such understanding leads to the learner developing a mental image of mathematical connectivity (Cambridge Mathematics Framework, 2015)
- connectivity: making important connections explicit in a consistent way that helps these connections to be referenced more easily, including those that may span multiple areas or otherwise tend to escape attention in existing curricula (Cambridge Mathematics Framework, 2018)

To write topic dependencies, we started with the specification to form expanded learning objectives (LOs). Next we identified the prior learning required and dependent topics. Once all the topics were completed, we put them on a spreadsheet and added each topic as a dependent topic for their prior topics and vice versa. We ended up with a summary for every topic, e.g. figure 1 for quadratics

Quadratics GCSE (general intro.) Multiply brackets (setting up completing the square) Simultaneous equations (general intro.) M2.1.2a Indices & Surds (exact roots)	Identify a quadratic function Factorise a quadratic with coefficient of x2 =1 Factorise a quadratic with coefficient of x2 ≠1 Derive the Quadratic formula Use the quadratic formula to solve quadratic equations Calculate the discriminant of a quadratic function Understand the conditions for real, repeated and no real roots. Recognise the nature of roots graphically Complete the square. Use completing the square to solve quadratic equations. Use completing the square to find the maximum or minimum value of a quadratic function & the line of symmetry. Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation. Find the points of intersection or the point of contact of a line and a curve. Make sketches relating to the above. Use Geogebra to investigate the above.	M2.1.2c Inequalities (Quadratic inequalities) M2.1.2d Polynomials (manipulation skills, simple polynomials) M2.1.2e Graphs & Graph Transformations (important function) M2.1.3b Geometry of circles (completing the square, solving line & circle) M2.1.5 Trigonometry (trig. quadratics) M2.1.7d Stationary Points (solve quadratics) M2.2.7b Constant acceleration formulæ (solving s=ut+½at2) M2.3.2a Simplify Rational Expressions (factorising expressions) M2.3.2f Functions in Modelling (quadratic models) M2.3.5c Compound angle formulae (quadratics) M2.3.7 Integration (find intersection points of curves) FM 2.1.2 Complex Numbers (solving quadratics) FM 2.1.4a Polynomials (identity: comparing coefficients; relationship between coefficients & roots) FM2.3.1a Momentum and Impulse (simultaneous equations) FM 2.4.8 Differential equations (auxiliary equation)

Figure 1. Quadratic dependencies from FMSP SoW

The Master Mind Map

Having completed the dependencies, we illustrated them on a Mind Map. This formed a beautiful diagram (figure 2), although it is difficult to read unless printed at A0 size!

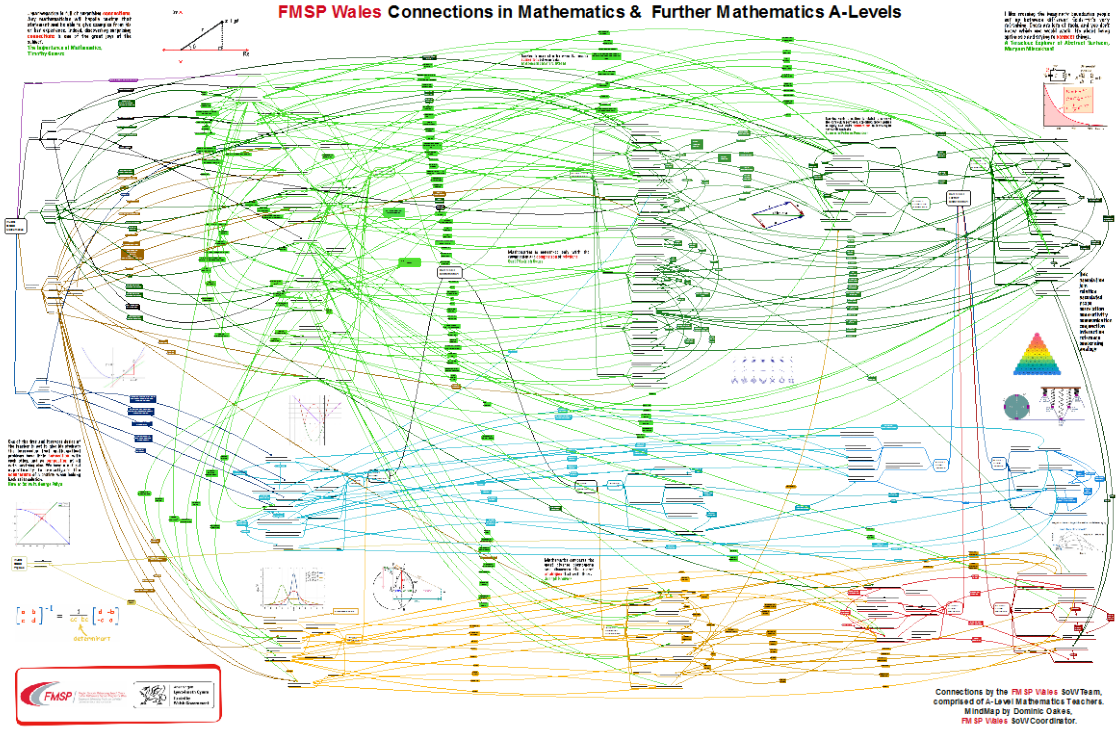

Figure 2. FMSP Mathematics and Further Mathematics Mind Map. The map shows all Specification Topics mapped from their papers and all the dependency connections. Green is pure, blue is statistics and orange/red is mechanics.

We can use this Mind Map to generate maps showing content connections. For example, for quadratic equations we have figure 3:

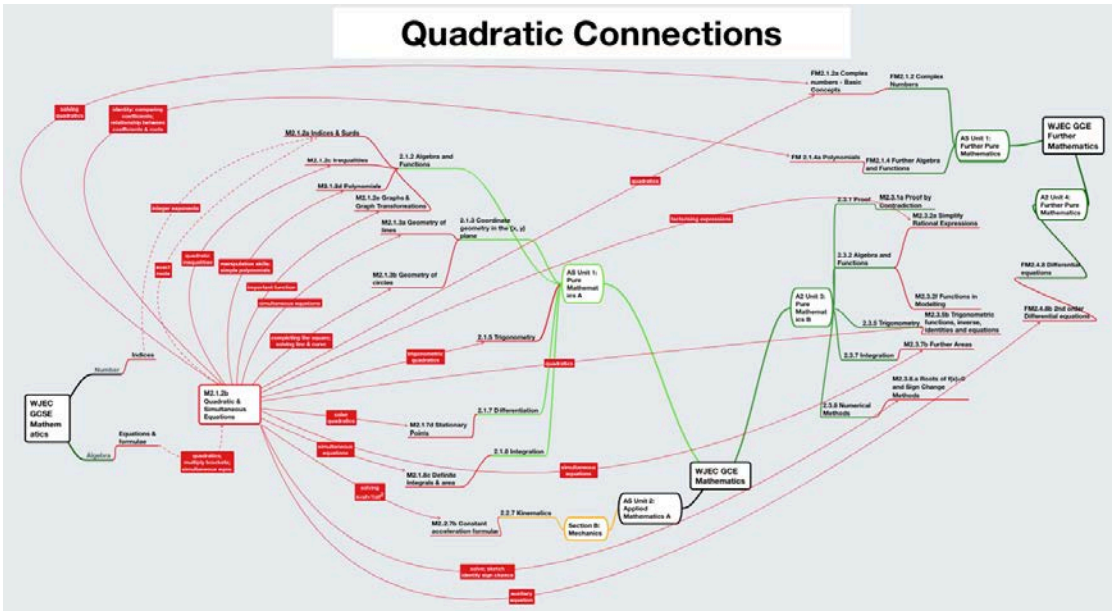

Figure 3. Quadratic equations in FMSP Mathematics and Further Mathematics

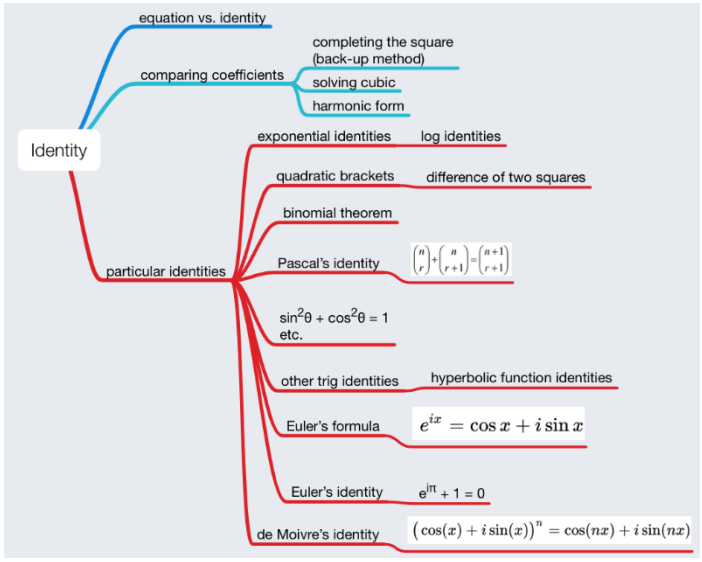

Figure 4. Identity

We are looking at ways to develop our map for wider functions such as clickability to resources. We also intend to add 'conceptual' functionality so that content using the same mathematical ideas can be identified (see figure 4).

BCME9 Workshop

Helen Hayes (HH, from the FMSP SoW team) spoke of her experience of working on the SoW. She mentioned the following as helpful to her planning:

- LOs helped understanding of the syllabus: discerning exactly what WJEC wanted, looked at similar strands from M AS to FM A2 – clarifying progression
- prior learning and dependencies: increased understanding of links from M to FM; connection to GCSE/Additional Mathematics; link the additional mathematics work to the new A level syllabus
- small topics impacted by/on a lot of others: not necessarily in the same area of mathematics; then linked forward – continuum
- connectivity and continuum: continuum and building blocks; I Like!; never having to unlearn things to learn a new model; can always go back to basics
- aim for students to make the connections: applying past knowledge; the "Oh, it's like …" and "we could use …" moments

- connections at A level filtered down: having seen the connections at A level, connections lower down the school became more obvious – I was looking for connections!; more connections (anchor points?) emphasised at KS3 and KS4
- team meetings to pull SoW together: process; meeting together to bring it all together – teams – discussing with others; working with others (FM can be lonely – only teacher at my school); creating something with a purpose.

Having introduced the idea of connection and the benefits of using them to design PoS, participants made topic maps (see figure 5). Whilst doing so, we discussed the possible impact of keeping connections central to our thinking regarding curriculum design. There were a number of interesting points from participants:

> P1: Can see how using connection will lead to more 'aha' moments and enthuse children
>
> P2: Difficulty of weaker students not being able to cope with multiple approaches – essentially, they can only cope with algorithms
>
> P3: As a tutor the 'standard approach' has generally not worked (which is why the student is coming for tutoring) so have to look for different ways into topics

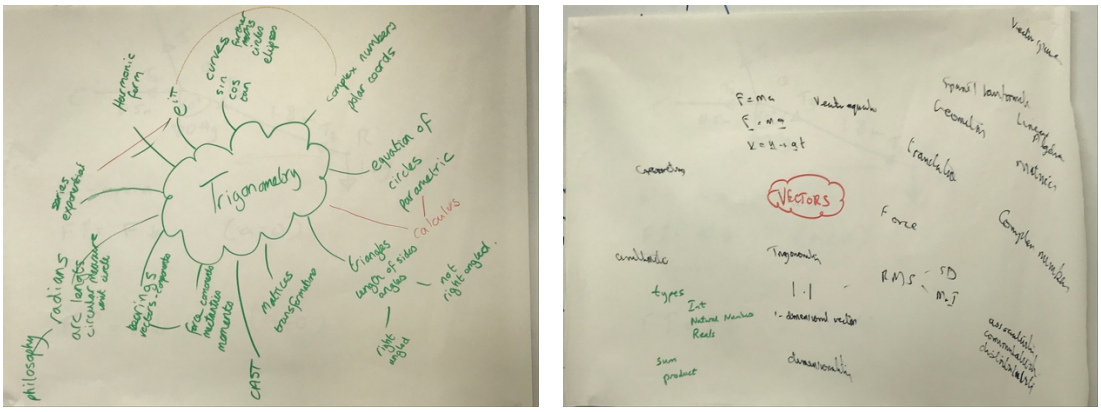

Figure 5. Trigonometry and Vectors Maps from BCME9 workshop

Paticipants comments focused on two main themes:

- how many connections there actually are
- how connections will now be emphasised explicitly.

Conclusions

This paper has shown how we have embedded connections into our SoW, which can lead to more explicitly connected PoS and more mathematically coherent teaching.

References

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