Algebraic division

Long division of polynomials

We looked at polynomial division in AS Mathematics. A reminder:

$$(x^3 + 5x^2 - 2x + 6) \div (x + 6)$$

$$- x + 4 \qquad r. -18$$

$$x + 6 \overline{)\, x^3 + 5x^2 - 2x + 6}$$

$$x^3 + 6x^2$$

So we have $(x^3 + 5x^2 - 2x + 6) \div (x + 6)$

$=$

$=$

We can express this result as:

$x^3 + 5x^2 - 2x + 6 =$

Simplify rational expression by division

So if we wished to simplify

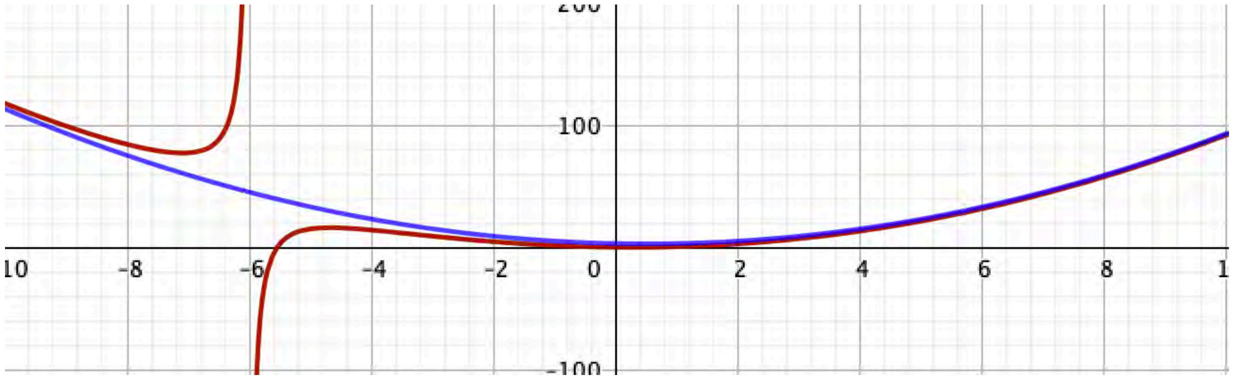
$$\frac{(x^3 + 5x^2 - 2x + 6)}{(x + 6)}$$

We could write it as

$$= x^2 - x + 4 - \frac{18}{x + 6}$$

This form has a number of advantages. It is easier to calculate values. It also allows us to sketch it more easily as the final term becomes very small as x becomes large

$$= x^2 - x + 4 - \frac{18}{x + 6}$$



Combined transformations

Reminder

If we have a function $y = f(x)$, there are 4 basic transformations:

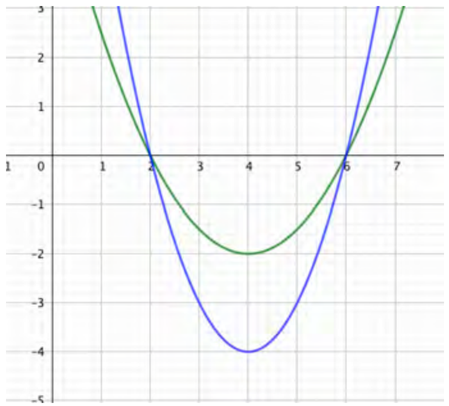
s.f. means scale factor

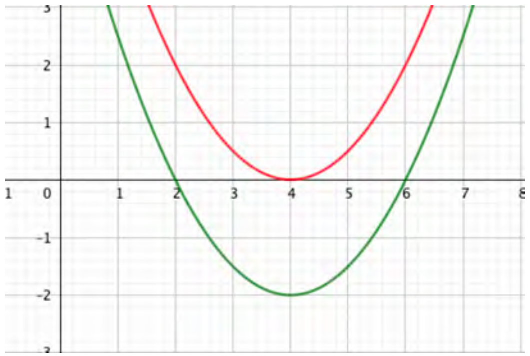
$y = af(x)$

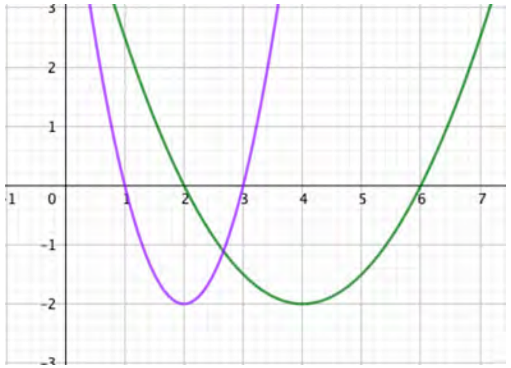
translation in y-direction of a units

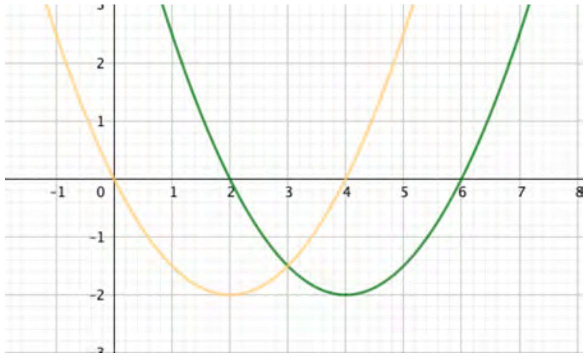
stretch from $x = 0$, s.f. $\frac{1}{a}$

$y = f(x + a)$









Combined transformations

Transformations in the opposite directions

If we carry out transformations in opposite directions,

For example, let us consider a stretch of s.f. 2 in the x-direction, and a translation of 3 in the y-direction.

The first takes $f(x)$ to $f\left(\frac{x}{2}\right)$

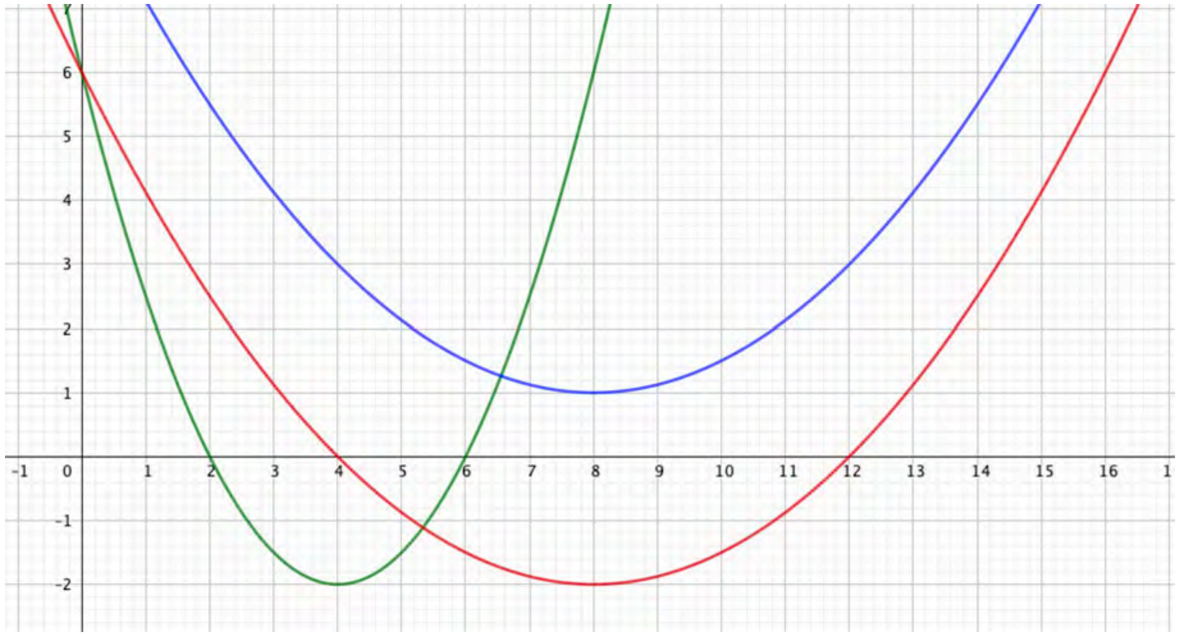
the second takes $f(x)$ to $f(x)+3$

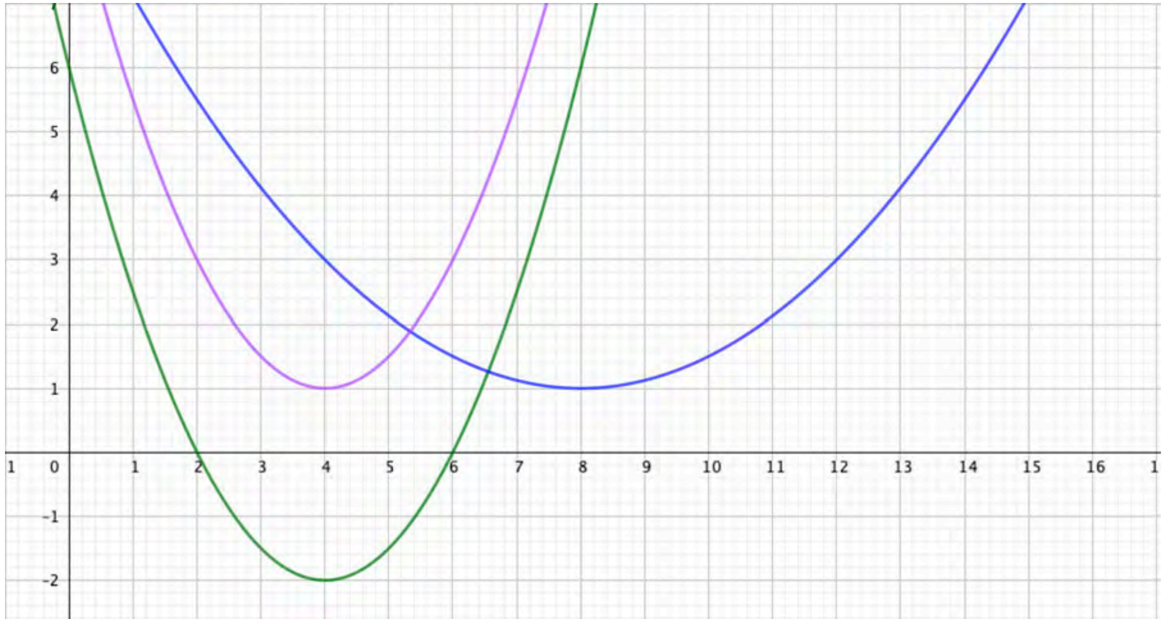
So we either have

$$f(x) \xrightarrow{stretch} \boxed{} \xrightarrow{translation} \boxed{}$$

or

$$f(x) \xrightarrow{translation} \boxed{} \xrightarrow{stretch} \boxed{}$$





Example 1 : a) Find the first three terms in the binomial expansion for $(1+9x)^{\frac{1}{3}}$, stating the values of x for which the series is valid.

b) By putting $x = 0.001$, find an estimate for the value of $\sqrt[3]{1009}$.

The binomial expansion for $(1+z)^{1/3}$ gives

$(1+z)^{1/3} =$

$=$ provided

Now put $z =$

$\Rightarrow \quad (1+9x)^{1/3} =$ provided

$\Rightarrow \quad (1+9x)^{1/3} =$

Putting $x = 0.001$

$\Rightarrow \quad (1.009)^{1/3} = \quad \approx \quad = 1.002991$

and $\sqrt[3]{1009} = \sqrt[3]{1000 \times 1.009} = \quad \approx$

Example 3: Find the first three non-zero terms in the expansion of $\sqrt{25 + x^2}$ and state the values of x for the expansion is valid.

Hence find a fraction which approximates the value of $\frac{1}{\sqrt{29}}$

$\sqrt{25+x^2} = (25+x^2)^{1/2} =$

$=$

$=$

$(1+z)^{1/2} =$

$=$

provided

so $\left(1+\frac{x^2}{25}\right)^{1/2} =$

provided

$=$

Example 3: Find the first three non-zero terms in the expansion of $\sqrt{25+x^2}$ and state the values of x for the expansion is valid.

Hence find a fraction which approximates the value of $\frac{1}{\sqrt{29}}$

$$\sqrt{25+x^2} = (25+x^2)^{1/2} = \left[25\left(1+\frac{x^2}{25}\right)\right]^{1/2}$$

$$= 25^{1/2}\left(1+\frac{x^2}{25}\right)^{1/2} = 5\left(1+\frac{x^2}{25}\right)^{1/2}$$

$$(1+z)^{1/2} = 1 + \frac{1}{2}z + \frac{\frac{1}{2}\times\left(\frac{1}{2}-1\right)}{2!}z^2 + \cdots\cdots$$

$$= 1 + \frac{1}{2}z - \frac{1}{8}z^2 + \cdots\cdots \quad \text{provided } -1 < z < 1$$

so $$\left(1+\frac{x^2}{25}\right)^{1/2} = 1 + \frac{1}{2}\left(\frac{x^2}{25}\right) - \frac{1}{8}\left(\frac{x^2}{25}\right)^2 + \cdots \quad \text{provided } -1 < \frac{x^2}{25} < 1$$

$$= 1 + \frac{1}{50}x^2 - \frac{1}{5000}x^4 + \cdots \quad \text{provided } -5 < x < 5$$

so $$\sqrt{25+x^2} = 5\left(1+\frac{x^2}{25}\right)^{1/2} = 5\left(1+\frac{1}{50}x^2 - \frac{1}{5000}x^4 + \cdots\right)$$

$$= 5 + \frac{1}{10}x^2 - \frac{1}{1000}x^4 + \cdots\cdots$$

provided $-5 < x < 5$

Putting $\boxed{}$ gives

$\sqrt{29} \approx \boxed{}$

$\Rightarrow \sqrt{29} \approx \frac{5384}{1000} = \frac{673}{125}$

$\Rightarrow \frac{1}{\sqrt{29}} \approx \boxed{}$