Starting Year 12 Summer school: Can you help me with my problems?

Croeso and Welcome

Please turn off your video and microphone.
We'll start at about 10am. Whilst waiting try:

How many of the whole numbers from 1 to 30 can you make using all the four numbers 1,2,3 and 4 once each, and any of the operations +,-,×,÷ or any brackets as many times as you want? e.g. $9 = 2 + 3 + 4 \times 1$

(We won't allow $9 = 23 - 14$*)*

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RhGMBC
Rhaglen Gymorth Mathemateg Bellach Cymru
Further Mathematics Support Programme Wales
FMSPW
Rheolir gan Brifysgol Abertawe, Sefydliad Gwyddorau Cyfrifiadurol a Mathemategol Cymru
Managed by Swansea University, Wales Institute of Mathematical and Computational Sciences
Ariennir gan Lywodraeth Cymru
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The Plan

- Plan for today: thinking mathematically and solving problems
- Key message: Don't panic! Being stuck is normal.
- Have a go and have fun.

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1,2,3,4, number

1 =

2 =

3 =

4 =

5 =

6 =

7 =

8 =

9 =

10 =

11 =

12 =

13 =

14 =

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A puzzle

1. Select any two digit number.
2. Add the digits together, and subtract from your number.
3. Using this final answer, look up the symbol in the table below.
4. Concentrate very hard on this symbol.

Source: www.datagenetics.com/blog/december52013

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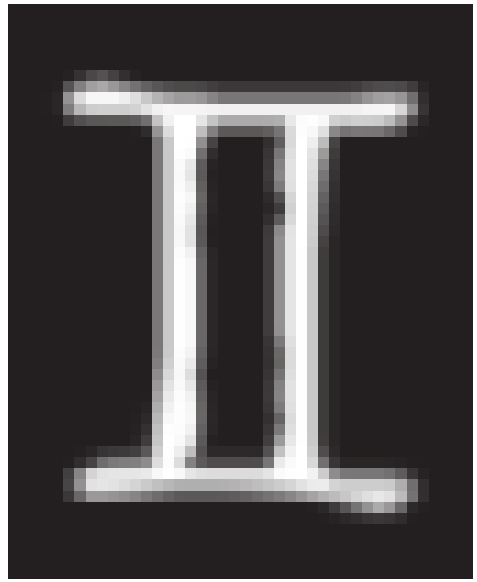
Your symbol was...



How does that work?

The first two instructions were

1. Select any two digit number.
2. Add the digits together, and subtract from your number.

Try these on 10 different two digit numbers: What do you notice?
Can you show that what you notice is always true?

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Showing it always works... and checking your arithmetic

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Divisibility tests for the connoisseur

Is it divisible by 19?
Remove the units digits, multiply it by two and add it to the remaining number. Keep repeating this process until you get a two digit number. If the two digit number is divisible by 19, so was the orginal e.g.

$$69768 \to 6976 + 2 \times 8 = 6992$$
$$6992 \to 699 + 2 \times 2 = 703$$
$$703 \to 70 + 2 \times 3 = 76$$

And $76 = 4 \times 19$, so yes.

Ones for you: what about 65113? what about 14878?

Note: changing the 2 to 5, -1, 4, -5 gives tests for 7, 11, 13 and 17 respectively.

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Hot dog, jumping frog

Have a go a swapping the frogs over at
https://nrich.maths.org/content/00/12/game1/frogs/

- Fewest moves to swap two blue frogs and two pink frogs?
- Fewest moves to swap three of each?
- Fewest moves to swap four of each?
- Fewest moves to swap five of each?
- Can you spot a pattern? Can you find a formula for the fewest moves with n of each? Can you show it's always true?

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Show your moves

n	2	3	4	5

Water pouring

You have three cups. One can hold 8 units, one 5 units and the third 3 units.

The 8 unit cup is full of water. Can you using the other cups, divide it into two lots of 4?

What about four cups holding 24, 13, 11 and 5 units. The 24 unit cup is full. Can you divide it into three lots of 8?

Census-taker problem

A census-taker knocks on a door and asks the woman inside how many children she has and how old they are.

I have three daughters, their ages are whole numbers, and the product of their ages is 36, says the mother.

That's not enough information, responds the census-taker.

I'd tell you the sum of their ages but you'd still be stumped.

I wish you'd tell me something more.

OK, my oldest daughter Annie likes dogs.

Perfect, thank you.

What are the ages of the three daughters?

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source: *The Art and Craft of Problem Solving* by Paul Zeitz

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Logic Grid and Nonogram Puzzles

There are logic grid puzzles available at
www.brainzilla.com/logic/logic-grid.

There are nonogram puzzles available at
www.puzzle-nonograms.com

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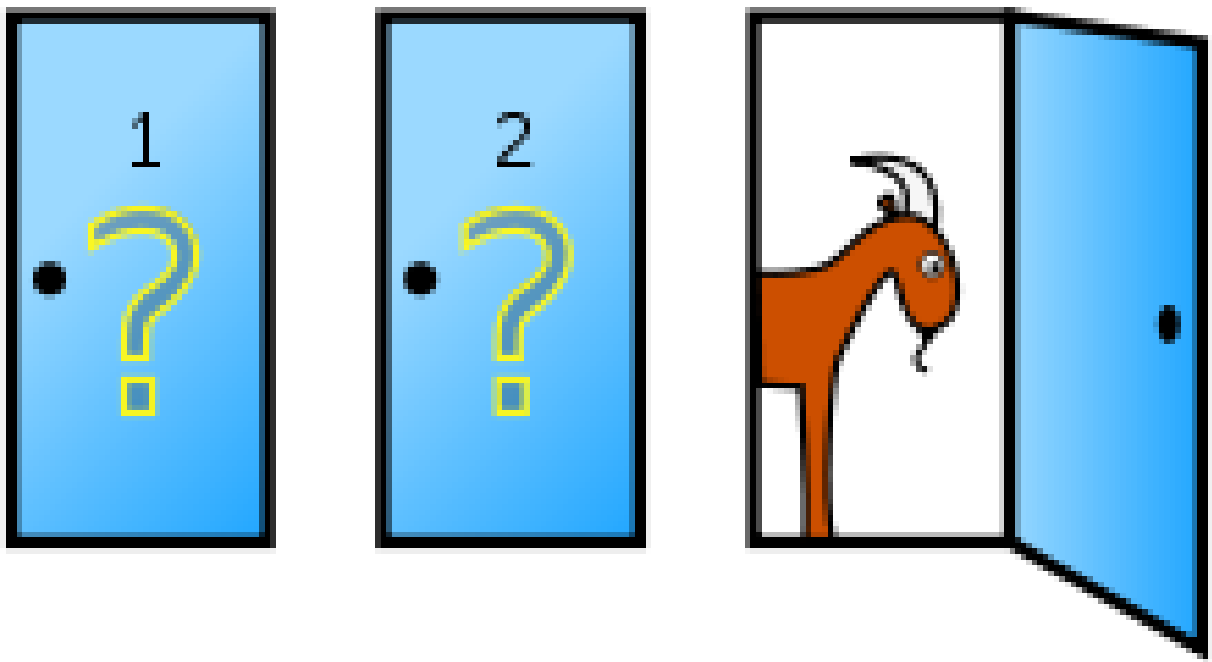
Socks in the sock drawer

A drawer contains red socks and black socks. When two socks are drawn at random, the probability that both are red is $\frac{1}{2}$. What's the least number of socks in the drawer? What's the least, if the number of black socks is even?

Source: *Fifty challenging problems in probability*, Frederick Mosteller

The Monty Hall problem

Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice?



Source: en.wikipedia.org/wiki/Monty_Hall_problem

Monty Hall analysis